6. Determinants

Exercise 6.1

1 A. Question

Write the minors and cofactors of each element of the first column of the following matrices and hence evaluate the determinant in each case:

$$A = \begin{bmatrix} 5 & 20 \\ 0 & -1 \end{bmatrix}$$

Answer

Let M_{ij} and C_{ij} represents the minor and co–factor of an element, where i and j represent the row and column.

The minor of the matrix can be obtained for a particular element by removing the row and column where the element is present. Then finding the absolute value of the matrix newly formed.

Also, $C_{ij} = (-1)^{i+j} \times M_{ij}$

$$A = \begin{bmatrix} 5 & 20 \\ 0 & -1 \end{bmatrix}$$

$M_{11} = -1$

$M_{21} = 20$

$C_{11} = (-1)^{1+1} \times M_{11}$

$= 1 \times -1$

$= -1$

$C_{21} = (-1)^{2+1} \times M_{21}$

$= 20 \times -1$

$= -20$

Now expanding along the first column we get

$|A| = a_{11} \times C_{11} + a_{21} \times C_{21}$

$= 5 \times (-1) + 0 \times (-20)$

$= -5$

1 B. Question

Write the minors and cofactors of each element of the first column of the following matrices and hence evaluate the determinant in each case:

$$A = \begin{bmatrix} -1 & 4 \\ 2 & 3 \end{bmatrix}$$

Answer

Let M_{ij} and C_{ij} represents the minor and co–factor of an element, where i and j represent the row and column.

The minor of matrix can be obtained for particular element by removing the row and column where the element is present. Then finding the absolute value of the matrix newly formed.

Also, $C_{ij} = (-1)^{i+j} \times M_{ij}$

$A = \begin{bmatrix} -1 & 4 \\ 2 & 3 \end{bmatrix}$

$M_{11} = 3$

$M_{21} = 4$

$C_{11} = (-1)^{1+1} \times M_{11}$

$= 1 \times 3$

$= 3$

$C_{21} = (-1)^{2+1} \times 4$

$= -1 \times 4$

$= -4$

Now expanding along the first column we get

$|A| = a_{11} \times C_{11} + a_{21} \times C_{21}$

$= -1 \times 3 + 2 \times (-4)$

$= -11$

1 C. Question

Write the minors and cofactors of each element of the first column of the following matrices and hence evaluate the determinant in each case:

$$A = \begin{bmatrix} 1 & -3 & 2 \\ 4 & -1 & 2 \\ 3 & 5 & 2 \end{bmatrix}$$

Answer

Let M_{ij} and C_{ij} represents the minor and co–factor of an element, where i and j represent the row and column.

The minor of the matrix can be obtained for a particular element by removing the row and column where the element is present. Then finding the absolute value of the matrix newly formed.

Also, $C_{ij} = (-1)^{i+j} \times M_{ij}$

$A = \begin{bmatrix} 1 & -3 & 2 \\ 4 & -1 & 2 \\ 3 & 5 & 2 \end{bmatrix}$

$\Rightarrow M_{11} = \begin{bmatrix} -1 & 2 \\ 5 & 2 \end{bmatrix}$

$M_{11} = -1 \times 2 - 5 \times 2$

$M_{11} = -12$

$\Rightarrow M_{21} = \begin{bmatrix} -3 & 2 \\ 5 & 2 \end{bmatrix}$

$M_{21} = -3 \times 2 - 5 \times 2$

$M_{21} = -16$

$\Rightarrow M_{31} = \begin{bmatrix} -3 & 2 \\ -1 & 2 \end{bmatrix}$

$M_{31} = -3 \times 2 - (-1) \times 2$

$M_{31} = -4$

$C_{11} = (-1)^{1+1} \times M_{11}$

$= 1 \times -12$

$= -12$

$C_{21} = (-1)^{2+1} \times M_{21}$

$= -1 \times -16$

$= 16$

$C_{31} = (-1)^{3+1} \times M_{31}$

$= 1 \times -4$

$= -4$

Now expanding along the first column we get

$|A| = a_{11} \times C_{11} + a_{21}\times C_{21}+ a_{31}\times C_{31}$

$= 1\times (-12) + 4 \times 16 + 3\times (-4)$

$= -12 + 64 -12$

$= 40$

1 D. Question

Write the minors and cofactors of each element of the first column of the following matrices and hence evaluate the determinant in each case:

$$A = \begin{bmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{bmatrix}$$

Answer

Let M_{ij} and C_{ij} represents the minor and co–factor of an element, where i and j represent the row and column.

The minor of the matrix can be obtained for a particular element by removing the row and column where the element is present. Then finding the absolute value of the matrix newly formed.

Also, $C_{ij} = (-1)^{i+j} \times M_{ij}$

$$A = \begin{bmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{bmatrix}$$

$$\Rightarrow M_{11} = \begin{bmatrix} b & ca \\ c & ab \end{bmatrix}$$

$M_{11} = b \times ab - c \times ca$

$M_{11} = ab^2 - ac^2$

$$\Rightarrow M_{21} = \begin{bmatrix} a & bc \\ c & ab \end{bmatrix}$$

$M_{21} = a \times ab - c \times bc$

$M_{21} = a^2b - c^2b$

$\Rightarrow M_{31} = \begin{bmatrix} a & bc \\ b & ca \end{bmatrix}$

$M_{31} = a \times ca - b \times bc$

$M_{31} = a^2c - b^2c$

$C_{11} = (-1)^{1+1} \times M_{11}$

$= 1 \times (ab^2 - ac^2)$

$= ab^2 - ac^2$

$C_{21} = (-1)^{2+1} \times M_{21}$

$= -1 \times (a^2b - c^2b)$

$= c^2b - a^2b$

$C_{31} = (-1)^{3+1} \times M_{31}$

$= 1 \times (a^2c - b^2c)$

$= a^2c - b^2c$

Now expanding along the first column we get

$|A| = a_{11} \times C_{11} + a_{21} \times C_{21} + a_{31} \times C_{31}$

$= 1\times (ab^2 - ac^2) + 1 \times (c^2b - a^2b) + 1\times (a^2c - b^2c)$

$= ab^2 - ac^2 + c^2b - a^2b + a^2c - b^2c$

1 E. Question

Write the minors and cofactors of each element of the first column of the following matrices and hence evaluate the determinant in each case:

$$A = \begin{bmatrix} 0 & 2 & 6 \\ 1 & 5 & 0 \\ 3 & 7 & 1 \end{bmatrix}$$

Answer

Let M_{ij} and C_{ij} represents the minor and co–factor of an element, where i and j represent the row and column.

The minor of matrix can be obtained for particular element by removing the row and column where the element is present. Then finding the absolute value of the matrix newly formed.

Also, $C_{ij} = (-1)^{i+j} \times M_{ij}$

$A = \begin{bmatrix} 0 & 2 & 6 \\ 1 & 5 & 0 \\ 3 & 7 & 1 \end{bmatrix}$

$\Rightarrow M_{11} = \begin{bmatrix} 5 & 0 \\ 7 & 1 \end{bmatrix}$

$M_{11} = 5\times1 - 7\times0$

$M_{11} = 5$

$\Rightarrow M_{21} = \begin{bmatrix} 2 & 6 \\ 7 & 1 \end{bmatrix}$

$M_{21} = 2\times1 - 7\times6$

$M_{21} = -40$

$\Rightarrow M_{31} = \begin{bmatrix} 2 & 6 \\ 5 & 0 \end{bmatrix}$

$M_{31} = 2\times0 - 5\times6$

$M_{31} = -30$

$C_{11} = (-1)^{1+1} \times M_{11}$

$= 1 \times 5$

$= 5$

$C_{21} = (-1)^{2+1} \times M_{21}$

$= -1 \times -40$

$= 40$

$C_{31} = (-1)^{3+1} \times M_{31}$

$= 1 \times -30$

$= -30$

Now expanding along the first column we get

$|A| = a_{11} \times C_{11} + a_{21}\times C_{21} + a_{31}\times C_{31}$

$= 0\times 5 + 1 \times 40 + 3\times (-30)$

$= 0 + 40 - 90$

$= 50$

1 F. Question

Write the minors and cofactors of each element of the first column of the following matrices and hence evaluate the determinant in each case:

$$A = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$$

Answer

Let M_{ij} and C_{ij} represents the minor and co–factor of an element, where i and j represent the row and column.

The minor of matrix can be obtained for particular element by removing the row and column where the element is present. Then finding the absolute value of the matrix newly formed.

Also, $C_{ij} = (-1)^{i+j} \times M_{ij}$

$$A = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$$

$\Rightarrow M_{11} = \begin{bmatrix} b & f \\ f & c \end{bmatrix}$

$M_{11} = b \times c - f \times f$

$M_{11} = bc - f^2$

$\Rightarrow M_{21} = \begin{bmatrix} h & g \\ f & c \end{bmatrix}$

$M_{21} = h \times c - f \times g$

$M_{21} = hc - fg$

$\Rightarrow M_{31} = \begin{bmatrix} h & g \\ b & f \end{bmatrix}$

$M_{31} = h \times f - b \times g$

$M_{31} = hf - bg$

$C_{11} = (-1)^{1+1} \times M_{11}$

$= 1 \times (bc - f^2)$

$= bc - f^2$

$C_{21} = (-1)^{2+1} \times M_{21}$

$= -1 \times (hc - fg)$

$= fg - hc$

$C_{31} = (-1)^{3+1} \times M_{31}$

$= 1 \times (hf - bg)$

$= hf - bg$

Now expanding along the first column we get

$|A| = a_{11} \times C_{11} + a_{21} \times C_{21} + a_{31} \times C_{31}$

$= a\times (bc - f^2) + h\times (fg - hc) + g\times (hf - bg)$

$= abc - af^2 + hgf - h^2c + ghf - bg^2$

1 G. Question

Write the minors and cofactors of each element of the first column of the following matrices and hence evaluate the determinant in each case:

$$A = \begin{bmatrix} 2 & -1 & 0 & 1 \\ -3 & 0 & 1 & -2 \\ 1 & 1 & -1 & 1 \\ 2 & -1 & 5 & 0 \end{bmatrix}$$

Answer

Let M_{ij} and C_{ij} represents the minor and co–factor of an element, where i and j represent the row and column.

The minor of matrix can be obtained for particular element by removing the row and column where the element is present. Then finding the absolute value of the matrix newly formed.

Also, $C_{ij} = (-1)^{i+j} \times M_{ij}$

$$A = \begin{bmatrix} 2 & -1 & 0 & 1 \\ -3 & 0 & 1 & -2 \\ 1 & 1 & -1 & 1 \\ 2 & -1 & 5 & 0 \end{bmatrix}$$

$\Rightarrow M_{11} = \begin{bmatrix} 0 & 1 & -2 \\ 1 & -1 & 1 \\ -1 & 5 & 0 \end{bmatrix}$

M_{11} = 0(-1×0 – 5×1) – 1(1×0 – (-1)×1) + (-2)(1×5 – (-1)×(-1))

M_{11} = - 9

$\Rightarrow M_{21} = \begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & 1 \\ -1 & 5 & 0 \end{bmatrix}$

M_{21} = -1(-1×0 – 5×1) – 0(1×0 – (-1)×1) + 1(1×5 – (-1)×(-1))

M_{21} = 9

$\Rightarrow M_{31} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & -2 \\ -1 & 5 & 0 \end{bmatrix}$

M_{31} = -1(1×0 – 5×(-2)) – 0(0×0 – (-1)×(-2)) + 1(0×5 – (-1)×1)

M_{31} = -9

$\Rightarrow M_{41} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & -2 \\ 1 & -1 & 1 \end{bmatrix}$

M_{41} = -1(1×1 – (-1)×(-2)) – 0(0×1 – 1×(-2)) + 1(0×(-1) – 1×1)

M_{41} = 0

$C_{11} = (-1)^{1+1} \times M_{11}$

= 1 × (-9)

= -9

$C_{21} = (-1)^{2+1} \times M_{21}$

= -1 × 9

= -9

$C_{31} = (-1)^{3+1} \times M_{31}$

= 1 × -9

= -9

$C_{41} = (-1)^{4+1} \times M_{41}$

= -1 × 0

= 0

Now expanding along the first column we get

$|A| = a_{11} \times C_{11} + a_{21} \times C_{21} + a_{31} \times C_{31} + a_{41} \times C_{41}$

= 2× (-9) + (-3) × -9 + 1× (-9) + 2× 0

= – 18 + 27 –9

= 0

2. Question

Evaluate the following determinants:

i. $\begin{vmatrix} x & -7 \\ x & 5x+1 \end{vmatrix}$

ii. $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$

iii. $\begin{vmatrix} \cos 15^\circ & -\sin 15^\circ \\ \sin 75^\circ & \cos 75^\circ \end{vmatrix}$

iv. $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$

Answer

I. Let $A = \begin{vmatrix} x & -7 \\ x & 5x+1 \end{vmatrix}$

⇒ |A| = x(5x + 1) – (–7)x

|A| = $5x^2 + 8x$

II. Let $A = \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$

⇒ |A| = cosθ × cosθ – (–sinθ) x sinθ

|A| = $\cos^2\theta + \sin^2\theta$

|A| = 1

III. Let $A = \begin{vmatrix} \cos 15^\circ & -\sin 15^\circ \\ \sin 75^\circ & \cos 75^\circ \end{vmatrix}$

⇒ |A| = cos15° × cos75° + sin15° x sin75°

|A| = cos(75 – 15)°

|A| = cos60°

|A| = 0.5.

IV. Let $A = \begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$

⇒ |A| = (a + ib)(a – ib) – (c + id)(–c + id)

= (a + ib)(a – ib) + (c + id)(c – id)

= $a^2 - i^2 b^2 + c^2 - i^2 d^2$

= $a^2 - (-1)b^2 + c^2 - (-1)d^2$

= $a^2 + b^2 + c^2 + d^2$

3. Question

Evaluate

$\begin{vmatrix} 2 & 3 & 7 \\ 13 & 17 & 5 \\ 15 & 20 & 12 \end{vmatrix}^2$

Answer

Since |AB|= |A||B|

$$|A| = \begin{vmatrix} 2 & 3 & 7 \\ 13 & 17 & 5 \\ 15 & 20 & 12 \end{vmatrix}$$

$$|A| = 2\begin{vmatrix} 17 & 5 \\ 20 & 12 \end{vmatrix} - 3\begin{vmatrix} 13 & 5 \\ 15 & 12 \end{vmatrix} + 7\begin{vmatrix} 13 & 17 \\ 15 & 20 \end{vmatrix}$$

= 2(17×12 – 5×20) – 3(13×12 – 5×15) + 7(13×20 – 15×17)

= 2(204 – 100) – 3(156 – 75) + 7(260 – 255)

= 2×104 – 3×81 + 7×5

= 208 – 243 +35

= 0

Now $|A|^2 = |A|\times|A|$

$|A|^2 = 0$

4. Question

Show that $\begin{vmatrix} \sin 10^\circ & -\cos 10^\circ \\ \sin 80^\circ & \cos 80^\circ \end{vmatrix} = 1$

Answer

Let $A = \begin{vmatrix} \sin 10^\circ & -\cos 10^\circ \\ \sin 80^\circ & \cos 80^\circ \end{vmatrix}$

Using sin(A+B) = sinA × cosB + cosA × sinB

⇒ |A| = sin10° × cos80° + cos10° x sin80°

|A| = sin(10 + 80)°

|A| = sin90°

|A| = 1

Hence Proved

5. Question

Evaluate $\begin{vmatrix} 2 & 3 & -5 \\ 7 & 1 & -2 \\ -3 & 4 & 1 \end{vmatrix}$ by two methods.

Answer

$$|A| = \begin{vmatrix} 2 & 3 & -5 \\ 7 & 1 & -2 \\ -3 & 4 & 1 \end{vmatrix}$$

I. Expanding along the first row

$$|A| = 2\begin{vmatrix} 1 & -2 \\ 4 & 1 \end{vmatrix} - 3\begin{vmatrix} 7 & -2 \\ -3 & 1 \end{vmatrix} - 5\begin{vmatrix} 7 & 1 \\ -3 & 4 \end{vmatrix}$$

= 2(1×1 – 4×(–2)) – 3(7×1 – (–2)×(–3)) – 5(7×4 – 1×(–3))

= 2(1 + 8) – 3(7 – 6) – 5(28 + 3)

= 2×9 – 3×1 – 5×31

= 18 – 3 – 155

= –140

II. Expanding along the second column

$$|A| = 2\begin{vmatrix}1 & -2\\4 & 1\end{vmatrix} - 7\begin{vmatrix}3 & -5\\4 & 1\end{vmatrix} - 3\begin{vmatrix}3 & -5\\1 & -2\end{vmatrix}$$

= 2(1×1 – 4×(–2)) – 7(3×1 – 4×(–5)) – 3(3×(–2) – 1×(–5))

= 2(1 + 8) – 7(3 + 20) – 3(–6 + 5)

= 2×9 – 7×23 – 3×(–1)

= 18 – 161 +3

= –140

6. Question

Evaluate : $\Delta = \begin{vmatrix}0 & \sin\alpha & -\cos\alpha\\ -\sin\alpha & 0 & \sin\beta\\ \cos\alpha & -\sin\beta & 0\end{vmatrix}$

Answer

$$\Delta = \begin{vmatrix}0 & \sin\alpha & -\cos\alpha\\ -\sin\alpha & 0 & \sin\beta\\ \cos\alpha & -\sin\beta & 0\end{vmatrix}$$

Expanding along the first row

$$|A| = 0\begin{vmatrix}0 & \sin\beta\\ -\sin\beta & 0\end{vmatrix} - \sin\alpha\begin{vmatrix}-\sin\alpha & \sin\beta\\ \cos\alpha & 0\end{vmatrix} - \cos\alpha\begin{vmatrix}-\sin\alpha & 0\\ \cos\alpha & -\sin\beta\end{vmatrix}$$

⇒ |A| = 0(0 – sinβ(–sinβ)) –sinα(–sinα× 0 – sinβ cosα) – cosα((–sinα)(–sinβ) – 0× cosα)

|A| = 0 + sinα sinβ cosα – cosα sinα sinβ

|A| = 0

7. Question

Evaluate :

$$\Delta = \begin{vmatrix}\cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha\\ -\sin\beta & \cos\beta & 0\\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha\end{vmatrix}$$

Answer

$$\Delta = \begin{vmatrix}\cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha\\ -\sin\beta & \cos\beta & 0\\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha\end{vmatrix}$$

Expanding along the second row

$$|A| = \sin\beta\begin{vmatrix}\cos\alpha\sin\beta & -\sin\alpha\\ \sin\alpha\sin\beta & \cos\alpha\end{vmatrix} + \cos\beta\begin{vmatrix}\cos\alpha\cos\beta & -\sin\alpha\\ \sin\alpha\cos\beta & \cos\alpha\end{vmatrix} - 0\begin{vmatrix}\cos\alpha\cos\beta & \cos\alpha\sin\beta\\ \sin\alpha\cos\beta & \sin\alpha\sin\beta\end{vmatrix}$$

⇒ |A| = sinβ (cosα×cosα sinβ + sinα × sinα sinβ) + cosβ (cosα cosβ × cosα + sinα×sinα cosβ) – 0

$|A| = \sin^2\beta\,(\cos^2\alpha + \sin^2\alpha) + \cos^2\beta\,(\cos^2\alpha + \sin^2\alpha)$

$|A| = \sin^2\beta\ (1) + \cos^2\beta\ (1)$

$|A| = \sin^2\beta + \cos^2\beta$

$|A| = 1$

8. Question

If $A = \begin{bmatrix} 2 & 5 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -3 \\ 2 & 5 \end{bmatrix}$, verify that |AB| = |A| |B|.

Answer

$A = \begin{bmatrix} 2 & 5 \\ 2 & 1 \end{bmatrix}$

$B = \begin{bmatrix} 4 & -3 \\ 2 & 5 \end{bmatrix}$

Now |A|= 2 × 1 – 2 × 5

|A|= 2 – 10

|A|= –8

Now |B|= 4 × 5 – 2 × (–3)

|B|= 20 + 6

|B|= 26

⇒ |A|×|B|= –8 × 26

|A|×|B|= –208

Now

$AB = \begin{bmatrix} 2 & 5 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 4 & -3 \\ 2 & 5 \end{bmatrix}$

$= \begin{bmatrix} 2 \times 4 + 5 \times 2 & 2 \times (-3) + 5 \times 5 \\ 2 \times 4 + 1 \times 2 & 2 \times (-3) + 1 \times 5 \end{bmatrix}$

$= \begin{bmatrix} 8 + 10 & -6 + 25 \\ 8 + 2 & -6 + 5 \end{bmatrix}$

$= \begin{bmatrix} 18 & 19 \\ 10 & -1 \end{bmatrix}$

|AB| = 18 × (–1) – 19 × 10

|AB| = –18 – 190

|AB| = –208

Hence |AB| =|A|×|B|.

9. Question

If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$, then show that $|3A| = 27|A|$.

Answer

$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{vmatrix}$

Expanding along the first row

$$|A| = 1\begin{vmatrix}1 & 2\\0 & 4\end{vmatrix} - 0\begin{vmatrix}0 & 2\\0 & 4\end{vmatrix} + 1\begin{vmatrix}0 & 1\\0 & 0\end{vmatrix}$$

= 1(1×4 – 2×0) – 0(0×4 – 0×2) + 1(0×0 – 0×1)

= 1(4 – 0) + 0 + 1(0 + 0)

= 1×4

= 4

Now

$$|3A| = \begin{vmatrix}3 & 0 & 3\\0 & 3 & 6\\0 & 0 & 12\end{vmatrix}$$

Expanding along the first row

$$|3A| = 3\begin{vmatrix}3 & 6\\0 & 12\end{vmatrix} - 0\begin{vmatrix}0 & 6\\0 & 12\end{vmatrix} + 3\begin{vmatrix}0 & 3\\0 & 0\end{vmatrix}$$

= 3(3×12 – 6×0) – 0(0×12 – 0×6) + 3(0×0 – 0×3)

= 3(36 – 0) + 0 + 3(0 + 0)

= 3×36

= 108

= 27 × 4

= 27 |A|

Hence, |3A|= 27 |A|

10 A. Question

Find the value of x, if

$$\begin{vmatrix}2 & 4\\5 & 1\end{vmatrix} = \begin{vmatrix}2x & 4\\6 & x\end{vmatrix}$$

Answer

$$\begin{vmatrix}2 & 4\\5 & 1\end{vmatrix} = \begin{vmatrix}2x & 4\\6 & x\end{vmatrix}$$

⇒ 2 × 1 – 4 × 5 = 2x × x – 4 × 6

⇒ 2 – 20 = $2x^2$ – 24

⇒ $2x^2$ = –18 +24

⇒ $2x^2$ = 6

⇒ x^2 = 3

⇒ x = ±√ 3

10 B. Question

Find the value of x, if

$$\begin{vmatrix}2 & 3\\4 & 5\end{vmatrix} = \begin{vmatrix}x & 3\\2x & 5\end{vmatrix}$$

Answer

$$\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$$

⇒ 2 × 5 – 4 × 3 = x × 5 – 2x × 3

⇒ 10 – 12 = 5x – 6x

⇒ –x = –2

⇒ x = 2

10 C. Question

Find the value of x, if

$$\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$$

Answer

$$\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$$

⇒ 3 × 1 – x × x = 3 × 1 – 4 × 2

⇒ 3 – x^2 = 3 – 8

⇒ – x^2 = –5 –3

⇒ –x^2 = –8

⇒ x = ±2√ 2

10 D. Question

Find the value of x, if

$$\begin{vmatrix} 3x & 7 \\ 2 & 4 \end{vmatrix} = 10$$

Answer

$$\begin{vmatrix} 3x & 7 \\ 2 & 4 \end{vmatrix} = 10$$

⇒ 3x × 4 – 7 × 2 = 10

⇒ 12x – 14 = 10

⇒ 12x= 10 +14

⇒ 12x = 24

⇒ x = 2

10 E. Question

Find the value of x, if

$$\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$$

Answer

$$\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$$

⇒ (x+1)(x+2) – (x–1)(x-3) = 4 × 3 – 1 × (–1)

$\Rightarrow (x^2 + 2x + x + 2) - (x^2 - 3x - x + 3) = 12 + 1$

$\Rightarrow -2x - 1 = 13$

$\Rightarrow -2x = 14$

$\Rightarrow x = -7$

10 F. Question

Find the value of x, if

$$\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & 5 \\ 8 & 3 \end{vmatrix}$$

Answer

$$\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & 5 \\ 8 & 3 \end{vmatrix}$$

$\Rightarrow 2x \times x - 5 \times 8 = 6 \times 3 - 5 \times 8$

$\Rightarrow 2x^2 - 40 = 18 - 40$

$\Rightarrow 2x^2 = 18$

$\Rightarrow x^2 = 9$

$\Rightarrow x = \pm 3$

11. Question

Find the integral value of x, if $\begin{vmatrix} x^2 & x & 1 \\ 0 & 2 & 1 \\ 3 & 1 & 4 \end{vmatrix} = 28$

Answer

$$|A| = \begin{vmatrix} x^2 & x & 1 \\ 0 & 2 & 1 \\ 3 & 1 & 4 \end{vmatrix}$$

Expanding along the first row

$$|A| = x^2 \begin{vmatrix} 2 & 1 \\ 1 & 4 \end{vmatrix} - x \begin{vmatrix} 0 & 1 \\ 3 & 4 \end{vmatrix} + 1 \begin{vmatrix} 0 & 2 \\ 3 & 1 \end{vmatrix}$$

$= x^2(2\times4 - 1\times1) - x(0\times4 - 1\times3) + 1(0\times1 - 2\times3)$

$= x^2(8 - 1) - x(0 - 3) + 1(0 - 6)$

$= 7x^2 + 3x - 6$

Also $|A| = 28$

$\Rightarrow 7x^2 + 3x - 6 = 28$

$\Rightarrow 7x^2 + 3x - 34 = 0$

$\Rightarrow 7x^2 + 17x - 14x - 34 = 0$

$\Rightarrow x(7x + 17) - 2(7x + 17) = 0$

$\Rightarrow (x - 2)(7x + 17) = 0$

$$x = 2, -\frac{17}{7}$$

Integer value of x is 2.

12 A. Question

For what value of x matrix A is singular?

$$A = \begin{bmatrix} 1+x & 7 \\ 3-x & 8 \end{bmatrix}$$

Answer

$|A| = 0$

$\begin{vmatrix} 1+x & 7 \\ 3-x & 8 \end{vmatrix} = 0$

⇒ (1 + x) × 8 – 7 × (3 – x) = 0

⇒ 8 + 8x – 21 + 7x = 0

⇒ 15x – 13 = 0

$\Rightarrow x = \frac{13}{15}$

12 B. Question

For what value of x matrix A is singular?

$$A = \begin{bmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{bmatrix}$$

Answer

$|A| = \begin{vmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{vmatrix}$

Expanding along the first row

$|A| = (x-1)\begin{vmatrix} x-1 & 1 \\ 1 & x-1 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & x-1 \end{vmatrix} + 1\begin{vmatrix} 1 & x-1 \\ 1 & 1 \end{vmatrix}$

= (x-1) ((x-1) (x-1)– 1×1) – 1((x-1) – 1×1) + 1(1×1 – 1×(x-1))

= (x-1) (x^2 – 2x + 1 – 1) – 1(x-1 – 1) + 1(1 – x+1)

= x(x-1) (x– 2) – 1(x-2) – (x-2)

= (x– 2) {x(x-1) – 1 – 1}

= (x– 2) (x^2 – x – 2)

For singular |A| = 0,

(x– 2) (x^2 – x – 2) = 0

(x– 2) (x^2 – 2x + x – 2) = 0

(x-2)(x-2)(x+1) = 0

∴ x = -1 or 2

Also |A| = 28

⇒ $7x^2$ + 3x – 6 =28

$\Rightarrow 7x^2 + 3x - 34 = 0$

$\Rightarrow 7x^2 + 17x - 14x - 34 = 0$

$\Rightarrow x(7x+ 17) - 2(7x +17) = 0$

$\Rightarrow (x-2)(7x +17) = 0$

Exercise 6.2

1 A. Question

Evaluate the following determinant:

$$\begin{vmatrix} 1 & 3 & 5 \\ 2 & 6 & 10 \\ 31 & 11 & 38 \end{vmatrix}$$

Answer

$$\text{Let}, \Delta = \begin{vmatrix} 1 & 3 & 5 \\ 2 & 6 & 10 \\ 31 & 11 & 38 \end{vmatrix} = 2\begin{vmatrix} 1 & 3 & 5 \\ 1 & 3 & 5 \\ 31 & 11 & 38 \end{vmatrix}$$

Applying, $R_2 \rightarrow R_2 - R_1$, we get,

$$\Rightarrow \Delta = 2\begin{vmatrix} 1 & 3 & 5 \\ 0 & 0 & 0 \\ 31 & 11 & 38 \end{vmatrix} = 0$$

So, $\Delta = 0$

1 B. Question

Evaluate the following determinant:

$$\begin{vmatrix} 67 & 19 & 21 \\ 39 & 13 & 14 \\ 81 & 24 & 26 \end{vmatrix}$$

Answer

$$\text{Let}, \Delta = \begin{vmatrix} 67 & 19 & 21 \\ 39 & 13 & 14 \\ 81 & 24 & 26 \end{vmatrix}$$

Applying, $C_1 \rightarrow C_1 - 4 C_3$, we get,

$$\Rightarrow \Delta = \begin{vmatrix} 4 & 19 & 21 \\ -3 & 13 & 14 \\ -3 & 24 & 26 \end{vmatrix}$$

Applying, $R_1 \rightarrow R_1 + R_2$ and $R_3 \rightarrow R_3 - R_2$, we get

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 32 & 35 \\ -3 & 13 & 14 \\ 0 & 11 & 12 \end{vmatrix}$$

Now, applying $R_2 \rightarrow R_2 + 3 R_1$, we get,

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 32 & 35 \\ 0 & 109 & 119 \\ 0 & 11 & 12 \end{vmatrix}$$

$= 1[(109)(12) - (119)(11)] = 1308 - 1309$

= – 1

So, Δ = – 1

1 C. Question

Evaluate the following determinant:

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$$

Answer

$$\text{Let}, \Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$$

= $a(bc - f^2) - h(hc - fg) + g(hf - bg)$

= $abc - af^2 - ch^2 + fgh + fgh - bg^2$

= $abc + 2fgh - af^2 - bg^2 - ch^2$

So, $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$

1 D. Question

Evaluate the following determinant:

$$\begin{vmatrix} 1 & -3 & 2 \\ 4 & -1 & 2 \\ 3 & 5 & 2 \end{vmatrix}$$

Answer

$$\text{Let}, \Delta = \begin{vmatrix} 1 & -3 & 2 \\ 4 & -1 & 2 \\ 3 & 5 & 2 \end{vmatrix}$$

$$\Rightarrow \Delta = 2\begin{vmatrix} 1 & -3 & 1 \\ 4 & -1 & 1 \\ 3 & 5 & 1 \end{vmatrix}$$

Applying, $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$, we get

$$\Rightarrow \Delta = 2\begin{vmatrix} 1 & -3 & 1 \\ 3 & 2 & 0 \\ 2 & 8 & 0 \end{vmatrix}$$

= 2[1(24 – 4)] = 40

So, Δ = 40

1 E. Question

Evaluate the following determinant:

$$\begin{vmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{vmatrix}$$

Answer

$$\text{Let, } \Delta = \begin{vmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{vmatrix}$$

Applying $C_3 \rightarrow C_3 - C_2$, we get,

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 4 & 5 \\ 4 & 9 & 7 \\ 9 & 16 & 9 \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 + C_1$, we get,

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 5 & 5 \\ 4 & 13 & 7 \\ 9 & 25 & 9 \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 - 5C_1$ and $C_3 \rightarrow C_3 - 5C_1$ we get,

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 0 & 0 \\ 4 & -7 & -13 \\ 9 & -20 & -36 \end{vmatrix}$$

$= 1[(- 7)(- 36) - (- 20)(- 13)] = 252 - 260$

$= - 8$

So, $\Delta = - 8$

1 F. Question

Evaluate the following determinant:

$$\begin{vmatrix} 6 & -3 & 2 \\ 2 & -1 & 2 \\ -10 & 5 & 2 \end{vmatrix}$$

Answer

$$\text{Let, } \Delta = \begin{vmatrix} 6 & -3 & 2 \\ 2 & -1 & 2 \\ -10 & 5 & 2 \end{vmatrix}$$

Applying, $R_1 \rightarrow R_1 - 3R_2$ and $R_3 \rightarrow R_3 + 5R_2$ we get,

$$\Rightarrow \Delta = \begin{vmatrix} 0 & 0 & -4 \\ 2 & -1 & 2 \\ 0 & 0 & 12 \end{vmatrix} = 0$$

So, $\Delta = 0$

1 G. Question

Evaluate the following determinant:

$$\begin{vmatrix} 1 & 3 & 9 & 27 \\ 3 & 9 & 27 & 1 \\ 9 & 27 & 1 & 3 \\ 27 & 1 & 3 & 9 \end{vmatrix}$$

Answer

$$\text{Let, } \Delta = \begin{vmatrix} 1 & 3 & 9 & 27 \\ 3 & 9 & 27 & 1 \\ 9 & 27 & 1 & 3 \\ 27 & 1 & 3 & 9 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 3 & 3^2 & 3^3 \\ 3 & 3^2 & 3^3 & 1 \\ 3^2 & 3^3 & 1 & 3 \\ 3^3 & 1 & 3 & 3^2 \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 + C_2 + C_3 + C_4$, we get,

$$\Rightarrow \Delta = \begin{vmatrix} 1 + 3 + 3^2 + 3^3 & 3 & 3^2 & 3^3 \\ 1 + 3 + 3^2 + 3^3 & 3^2 & 3^3 & 1 \\ 1 + 3 + 3^2 + 3^3 & 3^3 & 1 & 3 \\ 1 + 3 + 3^2 + 3^3 & 1 & 3 & 3^2 \end{vmatrix}$$

$$\Rightarrow \Delta = (1 + 3 + 3^2 + 3^3) \begin{vmatrix} 1 & 3 & 3^2 & 3^3 \\ 1 & 3^2 & 3^3 & 1 \\ 1 & 3^3 & 1 & 3 \\ 1 & 1 & 3 & 3^2 \end{vmatrix}$$

Now, applying $R_2 \rightarrow R_2 - R_1$, $R_3 \rightarrow R_3 - R_1$, $R_4 \rightarrow R_4 - R_1$, we get

$$\Rightarrow \Delta = (1 + 3 + 3^2 + 3^3) \begin{vmatrix} 1 & 3 & 3^2 & 3^3 \\ 0 & 3^2 - 3 & 3^3 - 3^2 & 1 - 3^3 \\ 0 & 3^3 - 3 & 1 - 3^2 & 3 - 3^3 \\ 0 & 1 - 3 & 3 - 3^2 & 3^2 - 3^3 \end{vmatrix}$$

$$\Rightarrow \Delta = (1 + 3 + 3^2 + 3^3) \begin{vmatrix} 6 & 18 & -26 \\ 24 & -8 & -24 \\ -2 & -6 & -18 \end{vmatrix}$$

$$\Rightarrow \Delta = (1 + 3 + 3^2 + 3^3)2^3 \begin{vmatrix} 3 & -9 & 13 \\ 12 & 4 & 12 \\ -1 & 3 & 9 \end{vmatrix}$$

Now, applying $R_1 \rightarrow R_1 + 3R_3$

$$\Rightarrow \Delta = (1 + 3 + 3^2 + 3^3)2^3 \begin{vmatrix} 0 & 0 & 40 \\ 12 & 4 & 12 \\ -1 & 3 & 9 \end{vmatrix}$$

$$= (1 + 3 + 3^2 + 3^3)2^3 \left[40\left(36 - (-4)\right)\right]$$

= (40)(8)(40)(40) = 512000

So, Δ = 512000

1 H. Question

Evaluate the following determinant:

$$\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$$

Answer

$$\text{Let}, \Delta = \begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$$

$$\Rightarrow \Delta = 6 \begin{vmatrix} 17 & 3 & 6 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$$

Applying $R_3 \rightarrow R_3 - R_1$, we get,

$$\Rightarrow \Delta = 6\begin{vmatrix} 17 & 3 & 6 \\ 1 & 3 & 4 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

So, $\Delta = 0$

2 A. Question

Without expanding, show that the value of each of the following determinants is zero:

$$\begin{vmatrix} 8 & 2 & 7 \\ 12 & 3 & 5 \\ 16 & 4 & 3 \end{vmatrix}$$

Answer

$$\text{Let}, \Delta = \begin{vmatrix} 8 & 2 & 7 \\ 12 & 3 & 5 \\ 16 & 4 & 3 \end{vmatrix}$$

Applying $R_3 \to R_3 - R_2$, we get

$$\Rightarrow \Delta = \begin{vmatrix} 8 & 2 & 7 \\ 12 & 3 & 5 \\ 4 & 1 & -2 \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$, we get

$$\Rightarrow \Delta = \begin{vmatrix} 8 & 2 & 7 \\ 4 & 1 & -2 \\ 4 & 1 & -2 \end{vmatrix}$$

As, $R_2 = R_3$, therefore the value of the determinant is zero.

2 B. Question

Without expanding, show that the value of each of the following determinants is zero:

$$\begin{vmatrix} 6 & -3 & 2 \\ 2 & -1 & 2 \\ -10 & 5 & 2 \end{vmatrix}$$

Answer

$$\text{Let}, \Delta = \begin{vmatrix} 6 & -3 & 2 \\ 2 & -1 & 2 \\ -10 & 5 & 2 \end{vmatrix}$$

Taking (– 2) common from C_1 we get,

$$\Rightarrow \Delta = \begin{vmatrix} -3 & -3 & 2 \\ -1 & -1 & 2 \\ 5 & 5 & 2 \end{vmatrix}$$

As, $C_1 = C_2$, hence the value of the determinant is zero.

2 C. Question

Without expanding, show that the value of each of the following determinants is zero:

$$\begin{vmatrix} 2 & 3 & 7 \\ 13 & 17 & 5 \\ 15 & 20 & 12 \end{vmatrix}$$

Answer

Let, $\Delta = \begin{vmatrix} 2 & 3 & 7 \\ 13 & 17 & 5 \\ 15 & 20 & 12 \end{vmatrix}$

Applying $C_3 \rightarrow C_3 - C_2$, gives

$\Rightarrow \Delta = \begin{vmatrix} 2 & 3 & 7 \\ 13 & 17 & 5 \\ 2 & 3 & 7 \end{vmatrix}$

As, $R_1 = R_3$, so value so determinant is zero.

2 D. Question

Without expanding, show that the value of each of the following determinants is zero:

$$\begin{vmatrix} 1/a & a^2 & bc \\ 1/b & b^2 & ac \\ 1/c & c^2 & ab \end{vmatrix}$$

Answer

Let, $\Delta = \begin{vmatrix} 1/a & a^2 & bc \\ 1/b & b^2 & ac \\ 1/c & c^2 & ab \end{vmatrix}$

Multiplying R_1, R_2 and R_3 with a, b and c respectively we get,

$\Rightarrow \Delta = \begin{vmatrix} 1 & a^3 & abc \\ 1 & b^3 & abc \\ 1 & c^3 & abc \end{vmatrix}$

Taking, abc common from C_3 gives,

$\Rightarrow \Delta = \begin{vmatrix} 1 & a^3 & 1 \\ 1 & b^3 & 1 \\ 1 & c^3 & 1 \end{vmatrix}$

As, $C_1 = C_3$ hence value of determinant is zero.

2 E. Question

Without expanding, show that the value of each of the following determinants is zero:

$$\begin{vmatrix} a+b & 2a+b & 3a+b \\ 2a+b & 3a+b & 4a+b \\ 4a+b & 5a+b & 6a+b \end{vmatrix}$$

Answer

Let, $\Delta = \begin{vmatrix} a+b & 2a+b & 3a+b \\ 2a+b & 3a+b & 4a+b \\ 4a+b & 5a+b & 6a+b \end{vmatrix}$

Applying $C_3 \rightarrow C_3 - C_2$, we get,

$\Rightarrow \Delta = \begin{vmatrix} a+b & 2a+b & a \\ 2a+b & 3a+b & a \\ 4a+b & 5a+b & a \end{vmatrix}$

Applying $C_2 \rightarrow C_2 - C_1$ gives,

$$\Rightarrow \Delta = \begin{vmatrix} a+b & a & a \\ 2a+b & a & a \\ 4a+b & a & a \end{vmatrix}$$

As, $C_2 = C_3$, so the value of the determinant is zero.

2 F. Question

Without expanding, show that the value of each of the following determinants is zero:

$$\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ac \\ 1 & c & c^2-ab \end{vmatrix}$$

Answer

$$\text{Let}, \Delta = \begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ac \\ 1 & c & c^2-ab \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & bc \\ 1 & b & ac \\ 1 & c & ab \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$, we get,

$$\Rightarrow \Delta = \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix} - \begin{vmatrix} 1 & a & bc \\ 0 & b-a & (a-b)c \\ 0 & c-a & (a-c)b \end{vmatrix}$$

Taking (b – a) and (c – a) common from R_2 and R_3 respectively,

$$\Rightarrow \Delta = (b-a)(c-a)\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b+a \\ 0 & 1 & c+a \end{vmatrix} - (b-a)(c-a)\begin{vmatrix} 1 & a & bc \\ 0 & 1 & -c \\ 0 & 1 & -b \end{vmatrix}$$

= [(b – a)(c – a)][(c + a) – (b + a) – (– b + c)]

= [(b – a)(c – a)][c + a + b – a – b – c]

= [(b – a)(c – a)][0] = 0

2 G. Question

Without expanding, show that the value of each of the following determinants is zero:

$$\begin{vmatrix} 49 & 1 & 6 \\ 39 & 7 & 4 \\ 26 & 2 & 3 \end{vmatrix}$$

Answer

$$\text{Let}, \Delta = \begin{vmatrix} 49 & 1 & 6 \\ 39 & 7 & 4 \\ 26 & 2 & 3 \end{vmatrix}$$

Applying, $C_1 \rightarrow C_1 - 8C_3$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 1 & 6 \\ 7 & 7 & 4 \\ 2 & 2 & 3 \end{vmatrix}$$

As, $C_1 = C_2$ hence, the determinant is zero.

2 H. Question

Without expanding, show that the value of each of the following determinants is zero:

$$\begin{vmatrix} 0 & x & y \\ -x & 0 & z \\ -y & -z & 0 \end{vmatrix}$$

Answer

$$\text{Let}, \Delta = \begin{vmatrix} 0 & x & y \\ -x & 0 & z \\ -y & -z & 0 \end{vmatrix}$$

Multiplying C_1, C_2 and C_3 with z, y and x respectively we get,

$$\Rightarrow \Delta = \left(\frac{1}{xyz}\right)\begin{vmatrix} 0 & xy & yx \\ -xz & 0 & zx \\ -yz & -zy & 0 \end{vmatrix}$$

Now, taking y, x and z common from R_1, R_2 and R_3 gives,

$$\Rightarrow \Delta = \left(\frac{1}{xyz}\right)\begin{vmatrix} 0 & x & x \\ -z & 0 & z \\ -y & -y & 0 \end{vmatrix}$$

Applying $C_2 \to C_2 - C_3$ gives,

$$\Rightarrow \Delta = \left(\frac{1}{xyz}\right)\begin{vmatrix} 0 & x & x \\ -z & -z & z \\ -y & -y & 0 \end{vmatrix}$$

As, $C_1 = C_2$, therefore determinant is zero.

2 I. Question

Without expanding, show that the value of each of the following determinants is zero:

$$\begin{vmatrix} 1 & 43 & 6 \\ 7 & 35 & 4 \\ 3 & 17 & 2 \end{vmatrix}$$

Answer

$$\text{Let}, \Delta = \begin{vmatrix} 1 & 43 & 6 \\ 7 & 35 & 4 \\ 3 & 17 & 2 \end{vmatrix}$$

Applying $C_2 \to C_2 - 7C_3$, we get

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 1 & 6 \\ 7 & 7 & 4 \\ 3 & 3 & 2 \end{vmatrix}$$

As, $C_1 = C_2$, hence determinant is zero.

2 J. Question

Without expanding, show that the value of each of the following determinants is zero:

$$\begin{vmatrix} 1^2 & 2^2 & 3^2 & 4^2 \\ 2^2 & 3^2 & 4^2 & 5^2 \\ 3^2 & 4^2 & 5^2 & 6^2 \\ 4^2 & 5^2 & 6^2 & 7^2 \end{vmatrix}$$

Answer

$$\text{Let}, \Delta = \begin{vmatrix} 1^2 & 2^2 & 3^2 & 4^2 \\ 2^2 & 3^2 & 4^2 & 5^2 \\ 3^2 & 4^2 & 5^2 & 6^2 \\ 4^2 & 5^2 & 6^2 & 7^2 \end{vmatrix}$$

Applying $C_3 \rightarrow C_3 - C_2$, and $C_4 \rightarrow C_4 - C_1$

$$\Rightarrow \Delta = \begin{vmatrix} 1^2 & 2^2 & 3^2 - 2^2 & 4^2 - 1^2 \\ 2^2 & 3^2 & 4^2 - 3^2 & 5^2 - 2^2 \\ 3^2 & 4^2 & 5^2 - 4^2 & 6^2 - 3^2 \\ 4^2 & 5^2 & 6^2 - 5^2 & 7^2 - 4^2 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1^2 & 2^2 & 5 & 15 \\ 2^2 & 3^2 & 7 & 21 \\ 3^2 & 4^2 & 9 & 27 \\ 4^2 & 5^2 & 11 & 33 \end{vmatrix}$$

Taking 3 common from C_4 we get,

$$\Rightarrow \Delta = 3\begin{vmatrix} 1^2 & 2^2 & 5 & 5 \\ 2^2 & 3^2 & 7 & 7 \\ 3^2 & 4^2 & 9 & 9 \\ 4^2 & 5^2 & 11 & 11 \end{vmatrix}$$

As, C3 = C4 so, the determinant is zero.

2 K. Question

Without expanding, show that the value of each of the following determinants is zero:

$$\begin{vmatrix} a & b & c \\ a+2x & b+2y & c+2z \\ x & y & z \end{vmatrix}$$

Answer

$$\text{Let}, \Delta = \begin{vmatrix} a & b & c \\ a + 2x & b + 2y & c + 2z \\ x & y & z \end{vmatrix}$$

Applying, $C_2 \rightarrow C_2 + C_1$ and $C_3 \rightarrow C_3 + C_1$

$$\Rightarrow \Delta = \begin{vmatrix} a & b & c \\ 2a + 2x & 2b + 2y & 2c + 2z \\ a + x & b + y & c + z \end{vmatrix}$$

Taking 2 common from R_2 we get,

$$\Rightarrow \Delta = 2\begin{vmatrix} a & b & c \\ a + x & b + y & c + z \\ a + x & b + y & c + z \end{vmatrix}$$

As, $R_2 = R_3$, hence value of determinant is zero.

2 L. Question

Without expanding, show that the value of each of the following determinants is zero:

$$\begin{vmatrix} (2^x+2^{-x})^2 & (2^x-2^{-x})^2 & 1 \\ (3^x+3^{-x})^2 & (3^x+3^{-x})^2 & 1 \\ (4^x+4^{-x})^2 & (4^x-4^{-x})^2 & 1 \end{vmatrix}$$

Answer

$$\text{Let}, \Delta = \begin{vmatrix} (2^x+2^{-x})^2 & (2^x-2^{-x})^2 & 1 \\ (3^x+3^{-x})^2 & (3^x-3^{-x})^2 & 1 \\ (4^x+4^{-x})^2 & (4^x-4^{-x})^2 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 2^{2x}+2^{-2x}+2 & 2^{2x}+2^{-2x}-2 & 1 \\ 3^{2x}+3^{-2x}+2 & 3^{2x}+3^{-2x}-2 & 1 \\ 4^{2x}+4^{-2x}+2 & 4^{2x}+4^{-2x}-2 & 1 \end{vmatrix}$$

Applying, $C_1 \to C_1 - C_2$, we get

$$\Rightarrow \Delta = \begin{vmatrix} 4 & 2^{2x}+2^{-2x}-2 & 1 \\ 4 & 3^{2x}+3^{-2x}-2 & 1 \\ 4 & 4^{2x}+4^{-2x}-2 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = 4\begin{vmatrix} 1 & 2^{2x}+2^{-2x}-2 & 1 \\ 1 & 3^{2x}+3^{-2x}-2 & 1 \\ 1 & 4^{2x}+4^{-2x}-2 & 1 \end{vmatrix}$$

As $C_1 = C_3$ hence determinant is zero.

2 M. Question

Without expanding, show that the value of each of the following determinants is zero:

$$\begin{vmatrix} \sin\alpha & \cos\alpha & \cos(\alpha+\delta) \\ \sin\beta & \cos\beta & \cos(\beta+\delta) \\ \sin\gamma & \cos\gamma & \cos(\gamma+\delta) \end{vmatrix}$$

Answer

$$\text{Let}, \Delta = \begin{vmatrix} \sin\alpha & \cos\alpha & \cos(\alpha+\delta) \\ \sin\beta & \cos\beta & \cos(\beta+\delta) \\ \sin\gamma & \cos\gamma & \cos(\gamma+\delta) \end{vmatrix}$$

Multiplying C_1 with $\sin\delta$, C_2 with $\cos\delta$, we get

$$\Rightarrow \Delta = \frac{1}{\sin\delta\cos\delta}\begin{vmatrix} \sin\alpha\sin\delta & \cos\alpha\cos\delta & \cos(\alpha+\delta) \\ \sin\beta\sin\delta & \cos\beta\cos\delta & \cos(\beta+\delta) \\ \sin\gamma\sin\delta & \cos\gamma\cos\delta & \cos(\gamma+\delta) \end{vmatrix}$$

Now, applying, $C_2 \to C_2 - C_1$, we get,

$$\Rightarrow \Delta = \frac{1}{\sin\delta\cos\delta}\begin{vmatrix} \sin\alpha\sin\delta & \cos\alpha\cos\delta-\sin\alpha\sin\delta & \cos(\alpha+\delta) \\ \sin\beta\sin\delta & \cos\beta\cos\delta-\sin\beta\sin\delta & \cos(\beta+\delta) \\ \sin\gamma\sin\delta & \cos\gamma\cos\delta-\sin\gamma\sin\delta & \cos(\gamma+\delta) \end{vmatrix}$$

$$\Rightarrow \Delta = \frac{1}{\sin\delta\cos\delta}\begin{vmatrix} \sin\alpha\sin\delta & \cos(\alpha+\delta) & \cos(\alpha+\delta) \\ \sin\beta\sin\delta & \cos(\beta+\delta) & \cos(\beta+\delta) \\ \sin\gamma\sin\delta & \cos(\gamma+\delta) & \cos(\gamma+\delta) \end{vmatrix}$$

As $C_2 = C_3$ hence determinant is zero.

2 N. Question

Without expanding, show that the value of each of the following determinants is zero:

$$\begin{vmatrix} \sin^2 23° & \sin^2 67° & \cos 180° \\ -\sin^2 67° & -\sin^2 23° & \cos^2 180° \\ \cos 180° & \sin^2 23° & \sin^2 67° \end{vmatrix}$$

Answer

$$\text{Let}, \Delta = \begin{vmatrix} \sin^2 23^\circ & \sin^2 67^\circ & \cos 180^\circ \\ -\sin^2 67^\circ & -\sin^2 23^\circ & \cos^2 180^\circ \\ \cos 180^\circ & \sin^2 23^\circ & \sin^2 67^\circ \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2$, we get

$$\Rightarrow \Delta = \begin{vmatrix} \sin^2 23^\circ + \sin^2 67^\circ & \sin^2 67^\circ & \cos 180^\circ \\ -\sin^2 67^\circ - \sin^2 23^\circ & -\sin^2 23^\circ & \cos^2 180^\circ \\ \cos 180^\circ + \sin^2 23^\circ & \sin^2 23^\circ & \sin^2 67^\circ \end{vmatrix}$$

Using, $\sin(90 - A) = \cos A$, $\sin^2 A + \cos^2 A = 1$,and $\cos 180^\circ = -1$,

$$\Rightarrow \Delta = \begin{vmatrix} \sin^2 23^\circ + \cos^2 23^\circ & \sin^2 67^\circ & \cos 180^\circ \\ -(\sin^2 67^\circ + \cos^2 67^\circ) & -\sin^2 23^\circ & \cos^2 180^\circ \\ -(1 - \sin^2 23^\circ) & \sin^2 23^\circ & \sin^2 67^\circ \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & \sin^2 67^\circ & -1 \\ -1 & -\sin^2 23^\circ & 1 \\ -\cos^2 23^\circ & \sin^2 23^\circ & \cos^2 23^\circ \end{vmatrix}$$

Taking, (– 1) common from C_1, we get

$$\Rightarrow \Delta = -\begin{vmatrix} -1 & \sin^2 67^\circ & -1 \\ 1 & -\sin^2 23^\circ & 1 \\ \cos^2 23^\circ & \sin^2 23^\circ & \cos^2 23^\circ \end{vmatrix}$$

Therefore, as $C_1 = C_3$ determinant is zero.

2 O. Question

Without expanding, show that the value of each of the following determinants is zero:

$$\begin{vmatrix} \cos(x+y) & -\sin(x+y) & \cos 2y \\ \sin x & \cos x & \sin y \\ -\cos x & \sin x & -\cos y \end{vmatrix}$$

Answer

$$\text{Let}, \Delta = \begin{vmatrix} \cos(x+y) & -\sin(x+y) & \cos 2y \\ \sin x & \cos x & \sin y \\ -\cos x & \sin x & -\cos y \end{vmatrix}$$

Multiplying R_2 with sin y and R_3 with cos y we get,

$$\Rightarrow \Delta = \frac{1}{\sin y \cos y}\begin{vmatrix} \cos(x+y) & -\sin(x+y) & \cos 2y \\ \sin x \sin y & \cos x \sin y & \sin^2 y \\ -\cos x \cos y & \sin x^2 \cos y & -\cos^2 y \end{vmatrix}$$

Now, applying $R_2 \to R_2 + R_3$, we get,

$$= \frac{1}{\sin y \cos y}\begin{vmatrix} \cos(x+y) & -\sin(x+y) & \cos 2y \\ \sin x \sin y - \cos x \cos y & \cos x \sin y + \sin x \cos y & \sin^2 y - \cos^2 y \\ -\cos x \cos y & \sin x \cos y & -\cos^2 y \end{vmatrix}$$

Taking (– 1) common from R_2, we get

$$= \frac{-1}{\sin y \cos y}\begin{vmatrix} \cos(x+y) & -\sin(x+y) & \cos 2y \\ -\sin x \sin y + \cos x \cos y & -(\cos x \sin y + \sin x \cos y) & -\sin^2 y + \cos^2 y \\ -\cos x \cos y & \sin x \cos y & -\cos^2 y \end{vmatrix}$$

$$\Rightarrow \Delta = \frac{-1}{\sin y \cos y}\begin{vmatrix} \cos(x+y) & -\sin(x+y) & \cos 2y \\ \cos(x+y) & -\sin(x+y) & \cos 2y \\ -\cos x \cos y & \sin x \cos y & -\cos^2 y \end{vmatrix}$$

As $R_1 = R_2$ hence determinant is zero.

2 P. Question

Without expanding, show that the value of each of the following determinants is zero:

$$\begin{vmatrix} \sqrt{23}+\sqrt{3} & \sqrt{5} & \sqrt{5} \\ \sqrt{15}+\sqrt{46} & 5 & \sqrt{10} \\ 3+\sqrt{115} & \sqrt{15} & 5 \end{vmatrix}$$

Answer

$$\text{Let, } \Delta = \begin{vmatrix} \sqrt{23}+\sqrt{3} & \sqrt{5} & \sqrt{5} \\ \sqrt{15}+\sqrt{46} & 5 & \sqrt{10} \\ 3+\sqrt{115} & \sqrt{15} & 5 \end{vmatrix}$$

Multiplying C_2 with $\sqrt{3}$ and C_3 with $\sqrt{23}$ we get,

$$\Rightarrow \Delta = \begin{vmatrix} \sqrt{23}+\sqrt{3} & \sqrt{15} & \sqrt{115} \\ \sqrt{15}+\sqrt{46} & 5\sqrt{3} & \sqrt{230} \\ 3+\sqrt{115} & \sqrt{45} & 5\sqrt{23} \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} \sqrt{23}+\sqrt{3} & \sqrt{5}(\sqrt{3}) & \sqrt{5}(\sqrt{23}) \\ \sqrt{15}+\sqrt{46} & \sqrt{5}(\sqrt{15}) & \sqrt{5}(\sqrt{46}) \\ 3+\sqrt{115} & \sqrt{5}(3) & \sqrt{5}(\sqrt{115}) \end{vmatrix}$$

Taking $\sqrt{5}$ common from C_2 and C_3 we get,

$$\Rightarrow \Delta = \sqrt{5}\sqrt{5}\begin{vmatrix} \sqrt{23}+\sqrt{3} & (\sqrt{3}) & (\sqrt{23}) \\ \sqrt{15}+\sqrt{46} & (\sqrt{15}) & (\sqrt{46}) \\ 3+\sqrt{115} & (3) & (\sqrt{115}) \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 + C_3$

$$\Rightarrow \Delta = 5\begin{vmatrix} \sqrt{23}+\sqrt{3} & \sqrt{23}+\sqrt{3} & (\sqrt{23}) \\ \sqrt{15}+\sqrt{46} & \sqrt{15}+\sqrt{46} & (\sqrt{46}) \\ 3+\sqrt{115} & 3+\sqrt{115} & (\sqrt{115}) \end{vmatrix}$$

As $C_1 = C_2$ hence determinant is zero.

2 Q. Question

Without expanding, show that the value of each of the following determinants is zero:

$$\begin{vmatrix} \sin^2 A & \cot A & 1 \\ \sin^2 B & \cot B & 1 \\ \sin^2 C & \cot C & 1 \end{vmatrix}$$, where A, B, C are the angles of ΔABC.

Answer

$$\text{Let}, \Delta = \begin{vmatrix} \sin^2 A & \cot A & 1 \\ \sin^2 B & \cot B & 1 \\ \sin^2 C & \cot C & 1 \end{vmatrix}$$

Now,

$\Delta = \sin^2 A (\cot B - \cot C) - \cot A (\sin^2 B - \sin^2 C) + 1 (\sin^2 B \cot C - \cot B \sin^2 C$

As A, B and C are angles of a triangle,

$A + B + C = 180°$

$\Delta = \sin^2 A \cot B - \sin^2 A \cot C - \cot A \sin^2 B + \cot A \sin^2 C + \sin^2 B \cot C - \cot B \sin^2 C$

By using formulae,

$$\frac{sinA}{a} = \frac{sinB}{b} = \frac{sinC}{c} = k$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2\,bc}, \cos B = \frac{a^2 + c^2 - b^2}{2\,a\,c}, \cos C = \frac{a^2 + b^2 - c^2}{2\,a\,b}$$

$\Delta = 0$

Hence, Proved.

3. Question

Evaluate the following:

$$\begin{vmatrix} a & b+c & a^2 \\ b & c+a & b^2 \\ c & a+b & c^2 \end{vmatrix}$$

Answer

$$\text{Let}, \Delta = \begin{vmatrix} a & b + c & a^2 \\ b & c + a & b^2 \\ c & a + b & c^2 \end{vmatrix}$$

Applying, $C_2 \to C_2 + C_1$

$$\Rightarrow \Delta = \begin{vmatrix} a & b + c + a & a^2 \\ b & c + a + b & b^2 \\ c & a + b + c & c^2 \end{vmatrix}$$

Taking, (a + b + c) common,

$$\Rightarrow \Delta = (a + b + c) \begin{vmatrix} a & 1 & a^2 \\ b & 1 & b^2 \\ c & 1 & c^2 \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$, and $R_3 \to R_3 - R_1$

$$\Rightarrow \Delta = (a + b + c) \begin{vmatrix} a & 1 & a^2 \\ b - a & 0 & b^2 - a^2 \\ c - a & 0 & c^2 - a^2 \end{vmatrix}$$

Taking, (b – c) and (c – a) common,

$$\Rightarrow \Delta = (a + b + c)(b - a)(c - a)\begin{vmatrix} a & 1 & a^2 \\ 1 & 0 & b + a \\ 1 & 0 & c + a \end{vmatrix}$$

= (a + b + c)(b – a)(c – a)(b – c)

So, Δ = (a + b + c)(b – a)(c – a)(b – c)

4. Question

Evaluate the following:

$$\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$

Answer

$$\text{Let}, \Delta = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$

Applying, $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$ we get,

$$\Rightarrow \Delta = \begin{vmatrix} 1 & a & bc \\ 0 & b - a & ca - bc \\ 0 & c - a & ab - bc \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a & bc \\ 0 & b - a & c(a - b) \\ 0 & c - a & b(a - c) \end{vmatrix}$$

Taking (a – b) and (a – c) common we get,

$$\Rightarrow \Delta = (a - b)(a - c)\begin{vmatrix} 1 & a & bc \\ 0 & -1 & c \\ 0 & -1 & b \end{vmatrix}$$

= (a – b)(c – a)(b – c)

So, Δ = (a – b)(b – c)(c – a)

5. Question

Evaluate the following:

$$\begin{vmatrix} x + \lambda & x & x \\ x & x + \lambda & x \\ x & x & x + \lambda \end{vmatrix}$$

Answer

$$\text{Let}, \Delta = \begin{vmatrix} x + \lambda & x & x \\ x & x + \lambda & x \\ x & x & x + \lambda \end{vmatrix}$$

Applying, $C_1 \rightarrow C_1 + C_2 + C_3$, we have,

$$\Rightarrow \Delta = \begin{vmatrix} 3x + \lambda & x & x \\ 3x + \lambda & x + \lambda & x \\ 3x + \lambda & x & x + \lambda \end{vmatrix}$$

Taking, (3x + λ) common, we get

$$\Rightarrow \Delta = (3x+\lambda)\begin{vmatrix} 1 & x & x \\ 1 & x+\lambda & x \\ 1 & x & x+\lambda \end{vmatrix}$$

Applying, $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$, we get,

$$\Rightarrow \Delta = (3x+\lambda)\begin{vmatrix} 1 & x & x \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix}$$

$= \lambda^2(3x + \lambda)$

So, $\Delta = \lambda^2(3x + \lambda)$

6. Question

Evaluate the following:

$$\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$$

Answer

$$\text{Let}, \Delta = \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$$

Applying, $C_1 \to C_1 + C_2 + C_3$, we get,

$$\Rightarrow \Delta = \begin{vmatrix} a+b+c & b & c \\ a+b+c & a & b \\ a+b+c & c & a \end{vmatrix}$$

Taking, (a + b + c) we get,

$$\Rightarrow \Delta = (a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & a & b \\ 1 & c & a \end{vmatrix}$$

Applying, $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$, we get,

$$\Rightarrow \Delta = (a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & a-b & b-c \\ 0 & c-b & a-c \end{vmatrix}$$

$= (a + b + c)[(a - b)(a - c) - (b - c)(c - b)]$

$= (a + b + c)[a^2 - ac - ab + bc + b^2 + c^2 - 2bc]$

$= (a + b + c)[a^2 + b^2 + c^2 - ac - ab - bc]$

So, $\Delta = (a + b + c)[a^2 + b^2 + c^2 - ac - ab - bc]$

7. Question

Evaluate the following:

$$\begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix}$$

Answer

Let, $\Delta = \begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix}$

Applying, $C_1 \to C_1 + C_2 + C_3$, we get,

$\Rightarrow \Delta = \begin{vmatrix} 2+x & 1 & 1 \\ 2+x & x & 1 \\ 2+x & 1 & x \end{vmatrix}$

$\Rightarrow \Delta = (2+x)\begin{vmatrix} 1 & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix}$

Applying, $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$, we get,

$\Rightarrow \Delta = (2+x)\begin{vmatrix} 1 & 1 & 1 \\ 0 & x-1 & 0 \\ 0 & 0 & x-1 \end{vmatrix}$

$= (2 + x)(x - 1)^2$

So, $\Delta = (2 + x)(x - 1)^2$

8. Question

Evaluate the following:

$$\begin{vmatrix} 0 & xy^2 & xz^2 \\ x^2y & 0 & yz^2 \\ x^2z & zy^2 & 0 \end{vmatrix}$$

Answer

Let, $\Delta = \begin{vmatrix} 0 & xy^2 & xz^2 \\ x^2y & 0 & yz^2 \\ x^2z & zy^2 & 0 \end{vmatrix}$

$= 0(0 - y^3z^3) - xy^2(0 - x^2yz^3) + xz^2(x^2y^3z - 0)$

$= 0 + x^3y^3z^3 + x^3y^3z^3$

$= 2x^3y^3z^3$

So, $\Delta = 2x^3y^3z^3$

9. Question

Evaluate the following:

$$\begin{vmatrix} 1+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix}$$

Answer

Let, $\Delta = \begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix}$

Applying $R_1 \to R_1 - R_2$ and $R_3 \to R_3 - R_2$

$$\Rightarrow \Delta = \begin{vmatrix} a & -a & 0 \\ x & a+y & z \\ 0 & -a & a \end{vmatrix}$$

Applying, $C_2 \to C_2 - C_1$

$$\Rightarrow \Delta = \begin{vmatrix} a & 0 & 0 \\ x & a+x+y & z \\ 0 & -a & a \end{vmatrix}$$

= a[a(a + x + y) + az] + 0 + 0

= $a^2(a + x + y + z)$

So, $\Delta = a^2(a + x + y + z)$

10. Question

If $\Delta = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}, \Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ yz & zx & xy \\ x & y & z \end{vmatrix}$, then prove that $\Delta + \Delta_1 = 0$.

Answer

$$\text{Let}, \Delta = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 1 \\ yz & zx & xy \\ x & y & z \end{vmatrix}$$

As $|A| = |A|^T$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + \begin{vmatrix} 1 & yz & x \\ 1 & zx & y \\ 1 & xy & z \end{vmatrix}$$

If any two rows or columns of the determinant are interchanged, then determinant changes its sign

$$\Rightarrow \Delta = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} - \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 0 & 0 & x^2 - yz \\ 0 & 0 & y^2 - zx \\ 0 & 0 & z^2 - xy \end{vmatrix} = 0$$

So, $\Delta = 0$

11. Question

Prove the following identities:

$$\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$$

Answer

$$\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix}$$

$$\text{L.H.S} = \begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix}$$

Apply $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} a+b+c & b & c \\ 0 & b-c & c-a \\ 2(a+b+c) & c+a & a+b \end{vmatrix}$$

Taking (a + b + c) common from C_1 we get,

$$= (a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & b-c & c-a \\ 2 & c+a & a+b \end{vmatrix}$$

Applying, $R_3 \to R_3 - 2R_1$

$$= (a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & b-c & c-a \\ 0 & c+a-2b & a+b-2c \end{vmatrix}$$

$= (a + b + c)[(b - c)(a + b - 2c) - (c - a)(c + a - 2b)]$

$= a^3 + b^3 + c^3 - 3abc$

As, L.H.S = R.H.S

Hence, proved.

12. Question

Prove the following identities:

$$\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} = 3abc - a^3 - b^3 - c^3$$

Answer

$$\text{L.H.S} = \begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix}$$

As $|A| = |A|^T$

$$\text{So, } \begin{vmatrix} b+c & c+a & a+b \\ a-b & b-c & c-a \\ a & b & c \end{vmatrix}$$

If any two rows or columns of the determinant are interchanged, then determinant changes its sign

$$-\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix}$$

Apply $C_1 \to C_1 + C_2 + C_3$

$$= -\begin{vmatrix} a+b+c & b & c \\ 0 & b-c & c-a \\ 2(a+b+c) & c+a & a+b \end{vmatrix}$$

Taking (a + b + c) common from C_1 we get,

$$= -(a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & b-c & c-a \\ 2 & c+a & a+b \end{vmatrix}$$

Applying, $R_3 \to R_3 - 2R_1$

$$= -(a + b + c)\begin{vmatrix} 1 & b & c \\ 0 & b-c & c-a \\ 0 & c+a-2b & a+b-2c \end{vmatrix}$$

= – (a + b + c)[(b – c)(a + b – 2c) – (c – a)(c + a – 2b)]

= 3abc – a^3 – b^3 – c^3

As, L.H.S = R.H.S, hence proved.

13. Question

Prove the following identities:

$$\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

Answer

$$\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$\text{L.H.S} = \begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix}$$

Applying, $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 2(a+b+c) & b+c & c+a \\ 2(a+b+c) & c+a & a+b \\ 2(a+b+c) & a+b & b+c \end{vmatrix}$$

$$= 2\begin{vmatrix} (a+b+c) & b+c & c+a \\ (a+b+c) & c+a & a+b \\ (a+b+c) & a+b & b+c \end{vmatrix}$$

Apply, $C_2 \to C_2 - C_1$, and $C_3 \to C_3 - C_1$, we have

$$= 2\begin{vmatrix} (a+b+c) & -a & -b \\ (a+b+c) & -b & -c \\ (a+b+c) & -c & -a \end{vmatrix}$$

$$= 2\begin{vmatrix} (a+b+c) & a & b \\ (a+b+c) & b & c \\ (a+b+c) & c & a \end{vmatrix}$$

$$= 2\left(\begin{vmatrix} c & a & b \\ a & b & c \\ b & c & a \end{vmatrix} + \begin{vmatrix} a & a & b \\ b & b & c \\ c & c & a \end{vmatrix} + \begin{vmatrix} b & a & b \\ c & b & c \\ a & c & a \end{vmatrix}\right)$$

$$= 2\begin{vmatrix} c & a & b \\ a & b & c \\ b & c & a \end{vmatrix}$$

$$= 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = \text{R.H.S}$$

Hence, proved.

14. Question

Prove the following identities:

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$$

Answer

$$\text{L.H.S} = \begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix},$$

R.H.S = $2(a + b + c)^2$

Applying $C_1 \to C_1 + C_2 + C_3$, we have

$$= \begin{vmatrix} 2(a+b+c) & a & b \\ 2(a+b+c) & b+c+2a & b \\ 2(a+b+c) & a & c+a+2b \end{vmatrix}$$

Taking, 2(a + b + c) common we get,

$$= 2(a+b+C) \begin{vmatrix} 1 & a & b \\ 1 & b+c+2a & b \\ 1 & a & c+a+2b \end{vmatrix}$$

Now, applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get,

$$= 2(a+b+C) \begin{vmatrix} 1 & a & b \\ 0 & b+c+a & 0 \\ 0 & 0 & c+a+b \end{vmatrix}$$

Thus, we have

L.H.S = $2(a + b + c)[1(a + b + c)^2]$

= $2(a + b + c)^3$ = R.H.S

Hence, proved.

15. Question

Prove the following identities:

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

Answer

$$\text{L.H.S} = \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Applying, $R_1 \to R_1 + R_2 + R_3$, we get,

$$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Taking (a + b + c) common we get,

$$= (a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$, we get,

$$= (a + b + c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -b-c-a & 0 \\ 2c & 0 & -c-a-b \end{vmatrix}$$

$$= (a + b + c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & b + c + a & 0 \\ 2c & 0 & b + c + a \end{vmatrix}$$

= (a + b + c)^3 = R.H.S

Hence, proved.

16. Question

Prove the following identities:

$$\begin{vmatrix} 1 & b+c & b^2+c^2 \\ 1 & c+a & c^2+a^2 \\ 1 & a+b & a^2+b^2 \end{vmatrix} = (a-b)(b-c)(c-a)$$

Answer

$$\text{L.H.S} = \begin{vmatrix} 1 & b + c & b^2 + c^2 \\ 1 & c + a & c^2 + a^2 \\ 1 & a + b & a^2 + b^2 \end{vmatrix}$$

Applying, $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get,

$$= \begin{vmatrix} 1 & b + c & b^2 + c^2 \\ 0 & a - b & a^2 - b^2 \\ 0 & a - c & a^2 - c^2 \end{vmatrix}$$

$$= (a-b)(a-c)\begin{vmatrix} 1 & b + c & b^2 + c^2 \\ 0 & 1 & a + b \\ 0 & 1 & a + c \end{vmatrix}$$

Applying $R_3 \to R_3 - R_2$, we get,

$$= (a-b)(a-c)\begin{vmatrix} 1 & b + c & b^2 + c^2 \\ 0 & 1 & a + b \\ 0 & 0 & c - a \end{vmatrix}$$

= (a – b)(a – c)(b – c) = R.H.S

Hence, proved.

17. Question

Prove the following identities:

$$\begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix} = 9(a+b)b^2$$

Answer

$$\text{L.H.S} = \begin{vmatrix} a & a + b & a + 2b \\ a + 2b & a & a + b \\ a + b & a + 2b & a \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$, we get,

$$= \begin{vmatrix} 3a+3b & 3a+3b & 3a+3b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix}$$

Taking, (3a + 2b) common we get,

$$= (3a+3b)\begin{vmatrix} 1 & 1 & 1 \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix}$$

Applying, $C_1 \to C_1 - C_2$ and $C_3 \to C_3 - C_2$, we get,

$$= (3a+3b)\begin{vmatrix} 0 & 1 & 0 \\ 2b & a & b \\ -b & a+2b & -2b \end{vmatrix}$$

$$= (3a+3b)b^2\begin{vmatrix} 0 & 1 & 0 \\ 2 & a & 1 \\ -1 & a+2b & -2 \end{vmatrix}$$

$= 3(a + b)b^2(3) = 9(a + b)\ b^2$

= R.H.S

Hence, proved.

18. Question

Prove the following identities:

$$\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

Answer

$$\mathrm{L.H.S} = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$

Applying, $R_1 \to a\ R_1$, $R_2 \to b\ R_2$, $R_3 \to c\ R_3$

$$= \left(\frac{1}{abc}\right)\begin{vmatrix} a & a^2 & abc \\ b & b^2 & cab \\ c & c^2 & abc \end{vmatrix}$$

$$= \left(\frac{abc}{abc}\right)\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}$$

$$= -\begin{vmatrix} a & 1 & a^2 \\ b & 1 & b^2 \\ c & 1 & c^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

Hence, proved.

19. Question

Prove the following identities:

$$\begin{vmatrix} z & x & y \\ z^2 & x^2 & y^2 \\ z^4 & x^4 & y^4 \end{vmatrix} = \begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^4 & y^4 & z^4 \end{vmatrix} = \begin{vmatrix} x^2 & y^2 & z^2 \\ x^4 & y^4 & z^4 \\ x & y & z \end{vmatrix} = xyz(x-y)(y-z)(z-x)(x+y+z)$$

Answer

$$\begin{vmatrix} z & x & y \\ z^2 & x^2 & y^2 \\ z^4 & x^4 & y^4 \end{vmatrix} = \begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^4 & y^4 & z^4 \end{vmatrix} = \begin{vmatrix} x^2 & y^2 & z^2 \\ x^4 & y^4 & z^4 \\ x & y & z \end{vmatrix}$$

$$= xyz(x-y)(y-z)(z-x)(x + y + z)$$

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^4 & y^4 & z^4 \end{vmatrix}$$

$$= xyz \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix}$$

$$= xyz \begin{vmatrix} 0 & 1 & 0 \\ x-y & y & z-y \\ x^3-y^3 & y^3 & z^3-y^3 \end{vmatrix}$$

$$= xyz(x-y)(z-y) \begin{vmatrix} 0 & 1 & 0 \\ 1 & y & 1 \\ x^2 + y^2 + xy & y^3 & z^2 + y^2 + zy \end{vmatrix}$$

$= - xyz(x - y)(z - y)[z^2 + y^2 + zy - x^2 - y^2 - xy]$

$= - xyz(x - y)(z - y)[(z - x)(z + x0 + y(z - x)]$

$= - xyz(x - y)(z - y)(z - x)(x + y + z)$

= R.H.S

Hence, proved.

20. Question

Prove the following identities:

$$\begin{vmatrix} (b+c)^2 & b^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)$$

$$(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$$

Answer

$$L.H.S = \begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix}$$

Applying, $C_1 \to C_1 + C_2 - 2C_3$

$$= \begin{vmatrix} (b+c)^2 - a^2 - 2bc & a^2 & bc \\ (c+a)^2 - b^2 - 2ca & b^2 & ca \\ (a+b)^2 - c^2 - 2ab & c^2 & ab \end{vmatrix}$$

$$= \begin{vmatrix} a^2 + b^2 + c^2 & a^2 & bc \\ a^2 + b^2 + c^2 & b^2 & ca \\ a^2 + b^2 + c^2 & c^2 & ab \end{vmatrix}$$

Taking $(a^2 + b^2 + c^2)$, common, we get,

$$= (a^2 + b^2 + c^2) \begin{vmatrix} 1 & a^2 & bc \\ 1 & b^2 & ca \\ 1 & c^2 & ab \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get,

$$= (a^2 + b^2 + c^2) \begin{vmatrix} 1 & a^2 & bc \\ 0 & b^2 - a^2 & ca - bc \\ 0 & c^2 - a^2 & ab - bc \end{vmatrix}$$

$$= (a^2 + b^2 + c^2)(b - a)(c - a) \begin{vmatrix} 1 & a^2 & bc \\ 0 & b + a & -c \\ 0 & c + a & -b \end{vmatrix}$$

$= (a^2 + b^2 + c^2)(b - a)(c - a)[(b + a)(- b) - (- c)(c + a)]$

$= (a^2 + b^2 + c^2)(a - b)(c - a)(b - c)(a + b + c)$

= R.H.S

Hence, proved.

21. Question

Prove the following identities:

$$\begin{vmatrix} (b+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix} = -2$$

Answer

$$L.H.S = \begin{vmatrix} (a + 1)(a + 2) & a + 2 & 1 \\ (a + 2)(a + 3) & a + 3 & 1 \\ (a + 3)(a + 4) & a + 4 & 1 \end{vmatrix}$$

Applying, $R_3 \to R_3 - R_2$

$$= \begin{vmatrix} (a + 1)(a + 2) & a + 2 & 1 \\ (a + 2)(a + 3) & a + 3 & 1 \\ (a + 3)2 & 1 & 0 \end{vmatrix}$$

Applying, $R_2 \to R_2 - R_1$

$$= \begin{vmatrix} (a + 1)(a + 2) & a + 2 & 1 \\ (a + 2)2 & 1 & 0 \\ (a + 3)2 & 1 & 0 \end{vmatrix}$$

$= [(2a + 4)(1) - (1)(2a + 6)]$

$= - 2$

= R.H.S

Hence, proved.

22. Question

Prove the following identities:

$$\begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$$

Answer

$$L.H.S = \begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c-(a-b)^2 & ab \end{vmatrix}$$

Applying, $C_2 \to C_2 - 2C_1 - 2C_3$, we get,

$$= \begin{vmatrix} a^2 & a^2-(b-c)^2-2a^2-2bc & bc \\ b^2 & b^2-(c-a)^2 a^2-(b-c)^2-2b^2-2ca & ca \\ c^2 & c-(a-b)^2 a^2-(b-c)^2-2c^2-2ab & ab \end{vmatrix}$$

$$= \begin{vmatrix} a^2 & -(a^2+b^2+c^2) & bc \\ b^2 & -(a^2+b^2+c^2) & ca \\ c^2 & -(a^2+b^2+c^2) & ab \end{vmatrix}$$

Taking, $-(a^2+b^2+c^2)$ common from C_2 we get,

$$= -(a^2+b^2+c^2)\begin{vmatrix} a^2 & 1 & bc \\ b^2 & 1 & ca \\ c^2 & 1 & ab \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$= -(a^2+b^2+c^2)\begin{vmatrix} a^2 & 1 & bc \\ b^2-a^2 & 0 & ca-bc \\ c^2-a^2 & 0 & ab-bc \end{vmatrix}$$

$$= -(a^2+b^2+c^2)(a-b)(c-a)\begin{vmatrix} a^2 & 1 & bc \\ -(b+a) & 0 & c \\ c+a & 0 & -b \end{vmatrix}$$

$= -(a^2+b^2+c^2)(a-b)(c-a)[(-(b+a))(-b)-(c)(c+a)]$

$= (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$

= R.H.S

Hence, proved.

23. Question

Prove the following identities:

$$\begin{vmatrix} 1 & a^2+bc & a^3 \\ 1 & b^2+ca & b^3 \\ 1 & c^2+ab & c^3 \end{vmatrix} = -(a-b)$$

$$(b-c)(c-a)(a^2+b^2+c^2)$$

Answer

$$L.H.S = \begin{vmatrix} 1 & a^2+bc & a^3 \\ 1 & b^2+ca & b^3 \\ 1 & c^2+ab & c^3 \end{vmatrix}$$

Applying, $R_2 \to R_2 - R_1$, and $R_3 \to R_3 - R_1$

$$= \begin{vmatrix} 1 & a^2 + bc & a^3 \\ 0 & b^2 + ca - a^2 - bc & b^3 - a^3 \\ 0 & c^2 + ab - a^2 - bc & c^3 - a^3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a^2 + bc & a^3 \\ 0 & b^2 - a^2 - c(b - a) & b^3 - a^3 \\ 0 & c^2 - a^2 + b(c - a) & c^3 - a^3 \end{vmatrix}$$

$$= (b - a)(c - a) \begin{vmatrix} 1 & a^2 + bc & a^3 \\ 0 & b + a - c & b^2 + a^2 + ab \\ 0 & c + a + b & c^2 + a^2 + ac \end{vmatrix}$$

$= (b - a)(c - a)[((b + a - c))(c^2 + a^2 + ac) - (b^2 + a^2 + ab)(c^2 + a^2 + ac)]$

$= -(a - b)(c - a)(b - c)(a^2 + b^2 + c^2)$

= R.H.S

Hence, proved.

24. Question

Prove the following identities:

$$\begin{vmatrix} a^2 & bc & ac + c^2 \\ a^2 + ab & b^2 & ac \\ ab & b^2 + bc & c^2 \end{vmatrix} = 4a^2b^2c^2$$

Answer

$$L.H.S = \begin{vmatrix} a^2 & bc & ac + c^2 \\ a^2 + ab & b^2 & ac \\ ab & b^2 + bc & c^2 \end{vmatrix}$$

Taking, a,b,c common from C_1, C_2, C_3 respectively we get,

$$= abc \begin{vmatrix} a & c & a + c \\ a + b & b & a \\ b & b + c & c \end{vmatrix}$$

Applying, $C_1 \to C_1 + C_2 + C_3$, we get,

$$= abc \begin{vmatrix} 2(a + c) & c & a + c \\ 2(a + b) & b & a \\ 2(b + c) & b + c & c \end{vmatrix}$$

$$= 2abc \begin{vmatrix} (a + c) & c & a + c \\ (a + b) & b & a \\ (b + c) & b + c & c \end{vmatrix}$$

Applying, $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$, we get,

$$= 2abc \begin{vmatrix} (a + c) & -a & 0 \\ (a + b) & -a & -b \\ (b + c) & 0 & -b \end{vmatrix}$$

Applying, $C_1 \to C_1 + C_2 + C_3$, we get,

$$= 2abc \begin{vmatrix} c & -a & 0 \\ 0 & -a & -b \\ c & 0 & -b \end{vmatrix}$$

Taking c, a, b common from C_1, C_2, C_3 respectively, we get,

$$= 2a^2b^2c^2\begin{vmatrix}1 & -1 & 0\\0 & -1 & -1\\1 & 0 & -1\end{vmatrix}$$

Applying, $R_3 \rightarrow R_3 - R_1$, we have

$$= 2a^2b^2c^2\begin{vmatrix}1 & -1 & 0\\0 & -1 & -1\\0 & 1 & -1\end{vmatrix}$$

$= 2a^2b^2c^2(2)$

$= 4a^2b^2c^2$ = R.H.S

Hence, proved.

25. Question

Prove the following identities:

$$\begin{vmatrix}x+4 & x & x\\x & x+4 & x\\x & x & x+4\end{vmatrix} = 16(3x+4)$$

Answer

$$\text{L.H.S} = \begin{vmatrix}x+4 & x & x\\x & x+4 & x\\x & x & x+4\end{vmatrix}$$

Applying, $C_1 \rightarrow C_1 + C_2 + C_3$, we get,

$$= \begin{vmatrix}3x+4 & x & x\\3x+4 & x+4 & x\\3x+4 & x & x+4\end{vmatrix}$$

Taking (3x + 4) common we get,

$$= (3x+4)\begin{vmatrix}1 & x & x\\1 & x+4 & x\\1 & x & x+4\end{vmatrix}$$

Applying, $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$, we get,

$$= (3x+4)\begin{vmatrix}1 & x & x\\0 & 4 & 0\\0 & 0 & 4\end{vmatrix}$$

= 16(3x + 4)

Hence proved.

26. Question

Prove the following identities –

$$\begin{vmatrix}1 & 1+p & 1+p+q\\2 & 3+2p & 4+3p+2q\\3 & 3+3p & 10+6p+3q\end{vmatrix} = 1$$

Answer

Let $\Delta = \begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q \end{vmatrix}$

Recall that the value of a determinant remains same if we apply the operation $R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$.

Applying $C_2 \to C_2 - pC_1$, we get

$$\Delta = \begin{vmatrix} 1 & 1+p-p(1) & 1+p+q \\ 2 & 3+2p-p(2) & 4+3p+2q \\ 3 & 6+3p-p(3) & 10+6p+3q \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 1 & 1+p+q \\ 2 & 3 & 4+3p+2q \\ 3 & 6 & 10+6p+3q \end{vmatrix}$$

Applying $C_3 \to C_3 - qC_1$, we get

$$\Delta = \begin{vmatrix} 1 & 1 & 1+p+q-q(1) \\ 2 & 3 & 4+3p+2q-q(2) \\ 3 & 6 & 10+6p+3q-q(3) \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 1 & 1+p \\ 2 & 3 & 4+3p \\ 3 & 6 & 10+6p \end{vmatrix}$$

Applying $C_3 \to C_3 - pC_2$, we get

$$\Delta = \begin{vmatrix} 1 & 1 & 1+p-p(1) \\ 2 & 3 & 4+3p-p(3) \\ 3 & 6 & 10+6p-p(6) \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 3 & 6 & 10 \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$, we get

$$\Delta = \begin{vmatrix} 1 & 1-1 & 1 \\ 2 & 3-2 & 4 \\ 3 & 6-3 & 10 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 0 & 1 \\ 2 & 1 & 4 \\ 3 & 3 & 10 \end{vmatrix}$$

Applying $C_3 \to C_3 - C_1$, we get

$$\Delta = \begin{vmatrix} 1 & 0 & 1-1 \\ 2 & 1 & 4-2 \\ 3 & 3 & 10-3 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 2 \\ 3 & 3 & 7 \end{vmatrix}$$

Expanding the determinant along R_1, we have

$\Delta = 1[(1)(7) - (3)(2)] - 0 + 0$

$\therefore \Delta = 7 - 6 = 1$

Thus, $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q \end{vmatrix} = 1$

27. Question

Prove the following identities –

$$\begin{vmatrix} a & b-c & c-b \\ a-c & b & c-a \\ a-b & b-a & c \end{vmatrix} = (a + b - c)(b + c - a)(c + a - b)$$

Answer

Let $\Delta = \begin{vmatrix} a & b-c & c-b \\ a-c & b & c-a \\ a-b & b-a & c \end{vmatrix}$

Recall that the value of a determinant remains same if we apply the operation $R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$.

Applying $R_1 \to R_1 - R_2$, we get

$$\Delta = \begin{vmatrix} a-(a-c) & b-c-(b) & c-b-(c-a) \\ a-c & b & c-a \\ a-b & b-a & c \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} c & -c & a-b \\ a-c & b & c-a \\ a-b & b-a & c \end{vmatrix}$$

Applying $R_1 \to R_1 - R_3$, we get

$$\Delta = \begin{vmatrix} c-(a-b) & -c-(b-a) & a-b-(c) \\ a-c & b & c-a \\ a-b & b-a & c \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} -a+b+c & a-b-c & a-b-c \\ a-c & b & c-a \\ a-b & b-a & c \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} -(a-b-c) & a-b-c & a-b-c \\ a-c & b & c-a \\ a-b & b-a & c \end{vmatrix}$$

Taking the term (a – b – c) common from R_1, we get

$$\Delta = (a-b-c)\begin{vmatrix} -1 & 1 & 1 \\ a-c & b & c-a \\ a-b & b-a & c \end{vmatrix}$$

Applying $C_2 \to C_2 + C_1$, we get

$$\Delta = (a-b-c)\begin{vmatrix} -1 & 1+(-1) & 1 \\ a-c & b+(a-c) & c-a \\ a-b & b-a+(a-b) & c \end{vmatrix}$$

$$\Rightarrow \Delta = (a-b-c)\begin{vmatrix} -1 & 0 & 1 \\ a-c & b+a-c & c-a \\ a-b & 0 & c \end{vmatrix}$$

Applying $C_3 \to C_3 + C_1$, we get

$$\Delta = (a-b-c)\begin{vmatrix} -1 & 0 & 1+(-1) \\ a-c & b+a-c & c-a+(a-c) \\ a-b & 0 & c+(a-b) \end{vmatrix}$$

$$\Rightarrow \Delta = (a-b-c)\begin{vmatrix} -1 & 0 & 0 \\ a-c & b+a-c & 0 \\ a-b & 0 & c+a-b \end{vmatrix}$$

Expanding the determinant along R_1, we have

Δ = (a – b – c)[–1(b + a – c)(c + a – b) – 0 + 0]

⇒ Δ = –(a – b – c)(b + a – c)(c + a – b)

∴ Δ = (b + c – a)(a + b – c)(c + a – b)

Thus, $\begin{vmatrix} a & b-c & c-b \\ a-c & b & c-a \\ a-b & b-a & c \end{vmatrix} = (a+b-c)(b+c-a)(c+a-b)$

28. Question

Prove the following identities –

$$\begin{vmatrix} a^2 & 2ab & b^2 \\ b^2 & a^2 & 2ab \\ 2ab & b^2 & a^2 \end{vmatrix} = \left(a^3+b^3\right)^2$$

Answer

Let $\Delta = \begin{vmatrix} a^2 & 2ab & b^2 \\ b^2 & a^2 & 2ab \\ 2ab & b^2 & a^2 \end{vmatrix}$

Recall that the value of a determinant remains same if we apply the operation $R_i \rightarrow R_i + kR_j$ or $C_i \rightarrow C_i + kC_j$.

Applying $R_1 \rightarrow R_1 + R_2$, we get

$$\Delta = \begin{vmatrix} a^2+b^2 & 2ab+a^2 & b^2+2ab \\ b^2 & a^2 & 2ab \\ 2ab & b^2 & a^2 \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 + R_3$, we get

$$\Delta = \begin{vmatrix} a^2+b^2+2ab & 2ab+a^2+b^2 & b^2+2ab+a^2 \\ b^2 & a^2 & 2ab \\ 2ab & b^2 & a^2 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} a^2+b^2+2ab & a^2+b^2+2ab & a^2+b^2+2ab \\ b^2 & a^2 & 2ab \\ 2ab & b^2 & a^2 \end{vmatrix}$$

Taking the term (a^2 + b^2 + 2ab) common from R_1, we get

$$\Delta = (a^2+b^2+2ab)\begin{vmatrix} 1 & 1 & 1 \\ b^2 & a^2 & 2ab \\ 2ab & b^2 & a^2 \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 - C_1$, we get

$$\Delta = (a+b)^2\begin{vmatrix} 1 & 1-1 & 1 \\ b^2 & a^2-b^2 & 2ab \\ 2ab & b^2-2ab & a^2 \end{vmatrix}$$

$$\Rightarrow \Delta = (a+b)^2\begin{vmatrix} 1 & 0 & 1 \\ b^2 & a^2-b^2 & 2ab \\ 2ab & b^2-2ab & a^2 \end{vmatrix}$$

Applying $C_3 \rightarrow C_3 - C_1$, we get

$$\Delta = (a+b)^2\begin{vmatrix} 1 & 0 & 1-1 \\ b^2 & a^2-b^2 & 2ab-b^2 \\ 2ab & b^2-2ab & a^2-2ab \end{vmatrix}$$

$$\Rightarrow \Delta = (a+b)^2 \begin{vmatrix} 1 & 0 & 0 \\ b^2 & a^2 - b^2 & 2ab - b^2 \\ 2ab & b^2 - 2ab & a^2 - 2ab \end{vmatrix}$$

Expanding the determinant along R_1, we have

$\Delta = (a + b)^2 [(a^2 - b^2)(a^2 - 2ab) - (b^2 - 2ab)(2ab - b^2)]$

$\Rightarrow \Delta = (a + b)^2 [a^4 - 2a^3b - b^2a^2 + 2ab^3 - 2ab^3 + b^4 + 4a^2b^2 - 2ab^3]$

$\Rightarrow \Delta = (a + b)^2 [a^4 - 2a^3b + 3a^2b^2 - 2ab^3 + b^4]$

$\Rightarrow \Delta = (a + b)^2 [a^4 + b^4 + 2a^2b^2 - 2a^3b - 2ab^3 + a^2b^2]$

$\Rightarrow \Delta = (a + b)^2 [(a^2 + b^2)^2 - 2ab(a^2 + b^2) + (ab)^2]$

$\Rightarrow \Delta = (a + b)^2 [(a^2 + b^2 - ab)^2]$

$\Rightarrow \Delta = [(a + b)(a^2 + b^2 - ab)]^2$

$\therefore \Delta = (a^3 + b^3)^2$

Thus, $\begin{vmatrix} a^2 & 2ab & b^2 \\ b^2 & a^2 & 2ab \\ 2ab & b^2 & a^2 \end{vmatrix} = (a^3 + b^3)^2$

29. Question

Prove the following identities –

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$$

Answer

Let $\Delta = \begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$

$$\Rightarrow \Delta = \begin{vmatrix} a\left(a+\frac{1}{a}\right) & ab & ac \\ ab & b\left(b+\frac{1}{b}\right) & bc \\ ca & cb & c\left(c+\frac{1}{c}\right) \end{vmatrix}$$

Taking a, b and c common from C_1, C_2 and C_3, we get

$$\Rightarrow \Delta = (abc) \begin{vmatrix} a+\frac{1}{a} & a & a \\ b & b+\frac{1}{b} & b \\ c & c & c+\frac{1}{c} \end{vmatrix}$$

Recall that the value of a determinant remains same if we apply the operation $R_i \rightarrow R_i + kR_j$ or $C_i \rightarrow C_i + kC_j$.

Applying $C_2 \rightarrow C_2 - C_1$, we get

$$\Delta = (abc)\begin{vmatrix} a+\frac{1}{a} & a-\left(a+\frac{1}{a}\right) & a \\ b & b+\frac{1}{b}-b & b \\ c & c-c & c+\frac{1}{c} \end{vmatrix}$$

$$\Rightarrow \Delta = (abc)\begin{vmatrix} a+\frac{1}{a} & -\frac{1}{a} & a \\ b & \frac{1}{b} & b \\ c & 0 & c+\frac{1}{c} \end{vmatrix}$$

Applying $C_3 \rightarrow C_3 - C_1$, we get

$$\Delta = (abc)\begin{vmatrix} a+\frac{1}{a} & -\frac{1}{a} & a-\left(a+\frac{1}{a}\right) \\ b & \frac{1}{b} & b-b \\ c & 0 & c+\frac{1}{c}-c \end{vmatrix}$$

$$\Rightarrow \Delta = (abc)\begin{vmatrix} a+\frac{1}{a} & -\frac{1}{a} & -\frac{1}{a} \\ b & \frac{1}{b} & 0 \\ c & 0 & \frac{1}{c} \end{vmatrix}$$

Multiplying a, b and c to R_1, R_2 and R_3, we get

$$\Delta = \begin{vmatrix} a\left(a+\frac{1}{a}\right) & a\left(-\frac{1}{a}\right) & a\left(-\frac{1}{a}\right) \\ b(b) & b\left(\frac{1}{b}\right) & 0 \\ c(c) & 0 & c\left(\frac{1}{c}\right) \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} a^2+1 & -1 & -1 \\ b^2 & 1 & 0 \\ c^2 & 0 & 1 \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 + R_2$, we get

$$\Delta = \begin{vmatrix} a^2+1+b^2 & -1+1 & -1+0 \\ b^2 & 1 & 0 \\ c^2 & 0 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1+a^2+b^2 & 0 & -1 \\ b^2 & 1 & 0 \\ c^2 & 0 & 1 \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 + R_3$, we get

$$\Delta = \begin{vmatrix} 1+a^2+b^2+c^2 & 0+0 & -1+1 \\ b^2 & 1 & 0 \\ c^2 & 0 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1+a^2+b^2+c^2 & 0 & 0 \\ b^2 & 1 & 0 \\ c^2 & 0 & 1 \end{vmatrix}$$

Expanding the determinant along R_1, we have

$\Delta = (1 + a^2 + b^2 + c^2)[(1)(1) - (0)(0)] - 0 + 0$

$\Rightarrow \Delta = (1 + a^2 + b^2 + c^2)(1)$

$\therefore \Delta = 1 + a^2 + b^2 + c^2$

Thus, $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$

30. Question

Prove the following identities –

$$\begin{vmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{vmatrix} = (a^3 - 1)^2$$

Answer

Let $\Delta = \begin{vmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{vmatrix}$

Recall that the value of a determinant remains same if we apply the operation $R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$.

Applying $R_1 \to R_1 + R_2$, we get

$$\Delta = \begin{vmatrix} 1+a^2 & a+1 & a^2+a \\ a^2 & 1 & a \\ a & a^2 & 1 \end{vmatrix}$$

Applying $R_1 \to R_1 + R_3$, we get

$$\Delta = \begin{vmatrix} 1+a^2+a & a+1+a^2 & a^2+a+1 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} a^2+a+1 & a^2+a+1 & a^2+a+1 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{vmatrix}$$

Taking the term $(a^2 + a + 1)$ common from R_1, we get

$$\Delta = (a^2+a+1)\begin{vmatrix} 1 & 1 & 1 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$, we get

$$\Delta = (a^2+a+1)\begin{vmatrix} 1 & 1-1 & 1 \\ a^2 & 1-a^2 & a \\ a & a^2-a & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = (a^2+a+1)\begin{vmatrix} 1 & 0 & 1 \\ a^2 & 1-a^2 & a \\ a & a^2-a & 1 \end{vmatrix}$$

Applying $C_3 \to C_3 - C_1$, we get

$$\Delta = (a^2+a+1)\begin{vmatrix} 1 & 0 & 1-1 \\ a^2 & 1-a^2 & a-a^2 \\ a & a^2-a & 1-a \end{vmatrix}$$

$$\Rightarrow \Delta = (a^2 + a + 1)\begin{vmatrix} 1 & 0 & 0 \\ a^2 & 1 - a^2 & a - a^2 \\ a & a^2 - a & 1 - a \end{vmatrix}$$

Expanding the determinant along R_1, we have

$\Delta = (a^2 + a + 1)(1)[(1 - a^2)(1 - a) - (a^2 - a)(a - a^2)]$

$\Rightarrow \Delta = (a^2 + a + 1)(1 - a - a^2 + a^3 - a^3 + a^4 + a^2 - a^3)$

$\Rightarrow \Delta = (a^2 + a + 1)(1 - a - a^3 + a^4)$

$\Rightarrow \Delta = (a^2 + a + 1)(a^4 - a^3 - a + 1)$

$\Rightarrow \Delta = (a^2 + a + 1)[a^3(a - 1) - (a - 1)]$

$\Rightarrow \Delta = (a^2 + a + 1)(a - 1)(a^3 - 1)$

$\Rightarrow \Delta = (a - 1)(a^2 + a + 1)(a^3 - 1)$

$\Rightarrow \Delta = (a^3 - 1)(a^3 - 1)$

$\therefore \Delta = (a^3 - 1)^2$

Thus, $\begin{vmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{vmatrix} = (a^3 - 1)^2$

31. Question

Prove the following identities –

$$\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -c \\ -b & -a & a+b+c \end{vmatrix} = 2(a + b)(b + c)(c + a)$$

Answer

Let $\Delta = \begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix}$

Recall that the value of a determinant remains same if we apply the operation $R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$.

Applying $R_1 \to R_1 + R_2$, we get

$$\Delta = \begin{vmatrix} a+b+c+(-c) & -c+(a+b+c) & -b+(-a) \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} a+b & a+b & -b-a \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix}$$

Applying $R_1 \to R_1 + R_3$, we get

$$\Delta = \begin{vmatrix} a+b+(-b) & a+b+(-a) & -b-a+(a+b+c) \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} a & b & c \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix}$$

Applying $C_2 \to C_2 + C_1$, we get

$$\Delta = \begin{vmatrix} a & b+a & c \\ -c & a+b+c+(-c) & -a \\ -b & -a+(-b) & a+b+c \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} a & b+a & c \\ -c & a+b & -a \\ -b & -(a+b) & a+b+c \end{vmatrix}$$

Applying $C_3 \rightarrow C_3 + C_1$, we get

$$\Delta = \begin{vmatrix} a & b+a & c+a \\ -c & a+b & -a+(-c) \\ -b & -(a+b) & a+b+c+(-b) \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} a & b+a & c+a \\ -c & a+b & -(a+c) \\ -b & -(a+b) & a+c \end{vmatrix}$$

Taking (a + b) and (c + a) common from C_2 and C_3, we get

$$\Delta = (a+b)(c+a)\begin{vmatrix} a & 1 & 1 \\ -c & 1 & -1 \\ -b & -1 & 1 \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 + R_1$, we get

$$\Delta = (a+b)(c+a)\begin{vmatrix} a & 1 & 1 \\ -c+a & 1+1 & -1+1 \\ -b & -1 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = (a+b)(c+a)\begin{vmatrix} a & 1 & 1 \\ -c+a & 2 & 0 \\ -b & -1 & 1 \end{vmatrix}$$

Applying $R_3 \rightarrow R_3 - R_1$, we get

$$\Delta = (a+b)(c+a)\begin{vmatrix} a & 1 & 1 \\ -c+a & 2 & 0 \\ -b-a & -1-1 & 1-1 \end{vmatrix}$$

$$\Rightarrow \Delta = (a+b)(c+a)\begin{vmatrix} a & 1 & 1 \\ -c+a & 2 & 0 \\ -b-a & -2 & 0 \end{vmatrix}$$

Expanding the determinant along C_3, we have

$\Delta = (a + b)(c + a)[(-c + a)(-2) - (-b - a)(2)]$

$\Rightarrow \Delta = (a + b)(c + a)[2c - 2a + 2a + 2b]$

$\Rightarrow \Delta = (a + b)(c + a)(2b + 2c)$

$\therefore \Delta = 2(a + b)(b + c)(c + a)$

Thus, $\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)(b+c)(c+a)$

32. Question

Prove the following identities –

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$$

Answer

Let $\Delta = \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$

Recall that the value of a determinant remains same if we apply the operation $R_i \rightarrow R_i + kR_j$ or $C_i \rightarrow C_i + kC_j$.

Applying $R_1 \rightarrow R_1 - R_2$, we get

$$\Delta = \begin{vmatrix} b+c-b & a-(c+a) & a-b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} c & -c & a-b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 - R_3$, we get

$$\Delta = \begin{vmatrix} c-c & -c-c & a-b-(a+b) \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 0 & -2c & -2b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 - C_1$, we get

$$\Delta = \begin{vmatrix} 0 & -2c-0 & -2b \\ b & c+a-b & b \\ c & c-c & a+b \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 0 & -2c & -2b \\ b & c+a-b & b \\ c & 0 & a+b \end{vmatrix}$$

Applying $C_3 \rightarrow C_3 - C_1$, we get

$$\Delta = \begin{vmatrix} 0 & -2c & -2b-0 \\ b & c+a-b & b-b \\ c & 0 & a+b-c \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 0 & -2c & -2b \\ b & c+a-b & 0 \\ c & 0 & a+b-c \end{vmatrix}$$

Expanding the determinant along R_1, we have

$\Rightarrow \Delta = 0 + (2c)[(b)(a + b - c)] + (-2b)[-(c)(c + a - b)]$

$\Rightarrow \Delta = 2bc(a + b - c) + 2bc(c + a - b)$

$\Rightarrow \Delta = 2bc[(a + b - c) + (c + a - b)]$

$\Rightarrow \Delta = 2bc[2a]$

$\therefore \Delta = 4abc$

Thus, $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$

33. Question

Prove the following identities –

$$\begin{vmatrix} b^2+c^2 & ab & ac \\ ba & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2$$

Answer

Let $\Delta = \begin{vmatrix} b^2+c^2 & ab & ac \\ ba & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix}$

Multiplying a, b and c to R_1, R_2 and R_3, we get

$$\Delta = \frac{1}{abc}\begin{vmatrix} a(b^2+c^2) & a(ab) & a(ac) \\ b(ba) & b(c^2+a^2) & b(bc) \\ c(ca) & c(cb) & c(a^2+b^2) \end{vmatrix}$$

$$\Rightarrow \Delta = \frac{1}{abc}\begin{vmatrix} a(b^2+c^2) & a^2b & a^2c \\ b^2a & b(c^2+a^2) & b^2c \\ c^2a & c^2b & c(a^2+b^2) \end{vmatrix}$$

Dividing C_1, C_2 and C_3 with a, b and c, we get

$$\Delta = \begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$

Recall that the value of a determinant remains same if we apply the operation $R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$.

Applying $R_1 \to R_1 - R_2$, we get

$$\Delta = \begin{vmatrix} b^2+c^2-b^2 & a^2-(c^2+a^2) & a^2-b^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} c^2 & -c^2 & a^2-b^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$

Applying $R_1 \to R_1 - R_3$, we get

$$\Delta = \begin{vmatrix} c^2-c^2 & -c^2-c^2 & a^2-b^2-(a^2+b^2) \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 0 & -2c^2 & -2b^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$, we get

$$\Delta = \begin{vmatrix} 0 & -2c^2-0 & -2b^2 \\ b^2 & c^2+a^2-b^2 & b^2 \\ c^2 & c^2-c^2 & a^2+b^2 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 0 & -2c^2 & -2b^2 \\ b^2 & c^2+a^2-b^2 & b^2 \\ c^2 & 0 & a^2+b^2 \end{vmatrix}$$

Applying $C_3 \to C_3 - C_1$, we get

$$\Delta = \begin{vmatrix} 0 & -2c^2 & -2b^2-0 \\ b^2 & c^2+a^2-b^2 & b^2-b^2 \\ c^2 & 0 & a^2+b^2-c^2 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 0 & -2c^2 & -2b^2 \\ b^2 & c^2+a^2-b^2 & 0 \\ c^2 & 0 & a^2+b^2-c^2 \end{vmatrix}$$

Expanding the determinant along R_1, we have

$\Delta = 0 + (2c^2)[(b^2)(a^2 + b^2 - c^2)] + (-2b^2)[-(c^2)(c^2 + a^2 - b^2)]$

$\Rightarrow \Delta = 2b^2c^2(a^2 + b^2 - c^2) + 2b^2c^2(c^2 + a^2 - b^2)$

$\Rightarrow \Delta = 2b^2c^2 [(a^2 + b^2 - c^2) + (c^2 + a^2 - b^2)]$

$\Rightarrow \Delta = 2b^2c^2[2a^2]$

$\therefore \Delta = 4a^2b^2c^2$

Thus, $\begin{vmatrix} b^2+c^2 & ab & ac \\ ba & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2$

34. Question

Prove the following identities –

$$\begin{vmatrix} 0 & b^2a & c^2a \\ a^2b & 0 & c^2b \\ a^2c & b^2c & 0 \end{vmatrix} = 2a^3b^3c^3$$

Answer

Let $\Delta = \begin{vmatrix} 0 & b^2a & c^2a \\ a^2b & 0 & c^2b \\ a^2c & b^2c & 0 \end{vmatrix}$

Taking a^2, b^2 and c^2 common from C_1, C_2 and C_3, we get

$$\Delta = (a^2b^2c^2)\begin{vmatrix} 0 & a & a \\ b & 0 & b \\ c & c & 0 \end{vmatrix}$$

Taking a, b and c common from R_1, R_2 and R_3, we get

$$\Rightarrow \Delta = (a^3b^3c^3)\begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

Recall that the value of a determinant remains same if we apply the operation $R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$.

Applying $C_2 \to C_2 - C_3$, we get

$$\Delta = (a^3b^3c^3)\begin{vmatrix} 0 & 1-1 & 1 \\ 1 & 0-1 & 1 \\ 1 & 1-0 & 0 \end{vmatrix}$$

$$\Rightarrow \Delta = (a^3b^3c^3)\begin{vmatrix} 0 & 0 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

Expanding the determinant along R_1, we have

$\Delta = (a^3b^3c^3)[0 - 0 + 1(1)(1) - (1)(-1)]$

$\Rightarrow \Delta = (a^3b^3c^3)[1 + 1]$

$\therefore \Delta = 2a^3b^3c^3$

Thus, $\begin{vmatrix} 0 & b^2a & c^2a \\ a^2b & c^2+a^2 & c^2b \\ a^2c & b^2c & 0 \end{vmatrix} = 2a^3b^3c^3$

35. Question

Prove the following identities –

$$\begin{vmatrix} \frac{a^2+b^2}{c} & c & c \\ a & \frac{b^2+c^2}{a} & a \\ b & b & \frac{c^2+a^2}{b} \end{vmatrix} = 4abc$$

Answer

Let $\Delta = \begin{vmatrix} \frac{a^2+b^2}{c} & c & c \\ a & \frac{b^2+c^2}{a} & a \\ b & b & \frac{c^2+a^2}{b} \end{vmatrix}$

Multiplying c, a and b to R_1, R_2 and R_3, we get

$$\Delta = \frac{1}{abc}\begin{vmatrix} a^2+b^2 & c^2 & c^2 \\ a^2 & b^2+c^2 & a^2 \\ b^2 & b^2 & c^2+a^2 \end{vmatrix}$$

Recall that the value of a determinant remains same if we apply the operation $R_i \rightarrow R_i + kR_j$ or $C_i \rightarrow C_i + kC_j$.

Applying $R_1 \rightarrow R_1 - R_2$, we get

$$\Delta = \frac{1}{abc}\begin{vmatrix} a^2+b^2-a^2 & c^2-(b^2+c^2) & c^2-a^2 \\ a^2 & b^2+c^2 & a^2 \\ b^2 & b^2 & c^2+a^2 \end{vmatrix}$$

$$\Rightarrow \Delta = \frac{1}{abc}\begin{vmatrix} b^2 & -b^2 & c^2-a^2 \\ a^2 & b^2+c^2 & a^2 \\ b^2 & b^2 & c^2+a^2 \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 - R_3$, we get

$$\Delta = \frac{1}{abc}\begin{vmatrix} b^2-b^2 & -b^2-b^2 & c^2-a^2-(c^2+a^2) \\ a^2 & b^2+c^2 & a^2 \\ b^2 & b^2 & c^2+a^2 \end{vmatrix}$$

$$\Rightarrow \Delta = \frac{1}{abc}\begin{vmatrix} 0 & -2b^2 & -2a^2 \\ a^2 & b^2+c^2 & a^2 \\ b^2 & b^2 & c^2+a^2 \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 - C_1$, we get

$$\Delta = \frac{1}{abc}\begin{vmatrix} 0 & -2b^2-0 & -2a^2 \\ a^2 & b^2+c^2-a^2 & a^2 \\ b^2 & b^2-b^2 & c^2+a^2 \end{vmatrix}$$

$$\Rightarrow \Delta = \frac{1}{abc}\begin{vmatrix} 0 & -2b^2 & -2a^2 \\ a^2 & b^2+c^2-a^2 & a^2 \\ b^2 & 0 & c^2+a^2 \end{vmatrix}$$

Applying $C_3 \rightarrow C_3 - C_1$, we get

$$\Delta = \frac{1}{abc}\begin{vmatrix} 0 & -2b^2 & -2a^2 - 0 \\ a^2 & b^2 + c^2 - a^2 & a^2 - a^2 \\ b^2 & 0 & c^2 + a^2 - b^2 \end{vmatrix}$$

$$\Rightarrow \Delta = \frac{1}{abc}\begin{vmatrix} 0 & -2b^2 & -2a^2 \\ a^2 & b^2 + c^2 - a^2 & 0 \\ b^2 & 0 & c^2 + a^2 - b^2 \end{vmatrix}$$

Expanding the determinant along R_1, we have

$$\Delta = \frac{1}{abc}\{0 + 2b^2[a^2(c^2 + a^2 - b^2)] - 2a^2[-b^2(b^2 + c^2 - a^2)]\}$$

$$\Rightarrow \Delta = \frac{1}{abc}\{2b^2a^2(c^2 + a^2 - b^2) + 2a^2b^2(b^2 + c^2 - a^2)\}$$

$$\Rightarrow \Delta = \frac{1}{abc}[2b^2a^2(c^2 + a^2 - b^2 + b^2 + c^2 - a^2)]$$

$$\Rightarrow \Delta = \frac{1}{abc}[2b^2a^2(2c^2)]$$

$$\Rightarrow \Delta = \frac{1}{abc}(4a^2b^2c^2)$$

$\therefore \Delta = 4abc$

Thus, $\begin{vmatrix} \frac{a^2+b^2}{c} & c & c \\ a & \frac{b^2+c^2}{a} & a \\ b & b & \frac{c^2+a^2}{b} \end{vmatrix} = 4abc$

36. Question

Prove the following identities –

$$\begin{vmatrix} -bc & b^2 + bc & c^2 + bc \\ a^2 + ac & -ac & c^2 + ac \\ a^2 + ab & b^2 + ab & -ab \end{vmatrix} = (ab + bc + ca)^3$$

Answer

Let $\Delta = \begin{vmatrix} -bc & b^2 + bc & c^2 + bc \\ a^2 + ac & -ac & c^2 + ac \\ a^2 + ab & b^2 + ab & -ab \end{vmatrix}$

Multiplying a, b and c to R_1, R_2 and R_3, we get

$$\Delta = \frac{1}{abc}\begin{vmatrix} -bc(a) & (b^2 + bc)a & (c^2 + bc)a \\ (a^2 + ac)b & (-ac)b & (c^2 + ac)b \\ (a^2 + ab)c & (b^2 + ab)c & (-ab)c \end{vmatrix}$$

$$\Rightarrow \Delta = \frac{1}{abc}\begin{vmatrix} -abc & ab^2 + abc & ac^2 + abc \\ a^2b + abc & -acb & bc^2 + abc \\ a^2c + abc & b^2c + abc & -abc \end{vmatrix}$$

Dividing C_1, C_2 and C_3 with a, b and c, we get

$$\Delta = \begin{vmatrix} -bc & ab + ac & ac + ab \\ ab + bc & -ac & bc + ab \\ ac + bc & bc + ac & -ab \end{vmatrix}$$

Recall that the value of a determinant remains same if we apply the operation $R_i \rightarrow R_i + kR_j$ or $C_i \rightarrow C_i + kC_j$.

Applying $R_1 \rightarrow R_1 + R_2$, we get

$$\Delta = \begin{vmatrix} -bc + (ab + bc) & ab + ac + (-ac) & ac + ab + (bc + ab) \\ ab + bc & -ac & bc + ab \\ ac + bc & bc + ac & -ab \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} ab & ab & 2ab + bc + ac \\ ab + bc & -ac & bc + ab \\ ac + bc & bc + ac & -ab \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 + R_3$, we get

$$\Delta = \begin{vmatrix} ab + (ac + bc) & ab + (bc + ac) & 2ab + bc + ac + (-ab) \\ ab + bc & -ac & bc + ab \\ ac + bc & bc + ac & -ab \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} ab + bc + ca & ab + bc + ca & ab + bc + ca \\ ab + bc & -ac & bc + ab \\ ac + bc & bc + ac & -ab \end{vmatrix}$$

Taking the term (a – b – c) common from R_1, we get

$$\Delta = (ab + bc + ca) \begin{vmatrix} 1 & 1 & 1 \\ ab + bc & -ac & bc + ab \\ ac + bc & bc + ac & -ab \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 - C_1$, we get

$$\Delta = (ab + bc + ca) \begin{vmatrix} 1 & 1 - 1 & 1 \\ ab + bc & -ac - (ab + bc) & bc + ab \\ ac + bc & bc + ac - (ac + bc) & -ab \end{vmatrix}$$

$$\Rightarrow \Delta = (ab + bc + ca) \begin{vmatrix} 1 & 0 & 1 \\ ab + bc & -(ab + bc + ca) & bc + ab \\ ac + bc & 0 & -ab \end{vmatrix}$$

Applying $C_3 \rightarrow C_3 - C_1$, we get

$$\Delta = (ab + bc + ca) \begin{vmatrix} 1 & 0 & 1 - 1 \\ ab + bc & -(ab + bc + ca) & bc + ab - (ab + bc) \\ ac + bc & 0 & -ab - (ac + bc) \end{vmatrix}$$

$$\Rightarrow \Delta = (ab + bc + ca) \begin{vmatrix} 1 & 0 & 0 \\ ab + bc & -(ab + bc + ca) & 0 \\ ac + bc & 0 & -(ab + bc + ca) \end{vmatrix}$$

Expanding the determinant along R_1, we have

$\Delta = (ab + bc + ca)(1)[(ab + bc + ca)(ab + bc + ca)]$

$\therefore \Delta = (ab + bc + ca)^3$

Thus, $\begin{vmatrix} -bc & b^2 + bc & c^2 + bc \\ a^2 + ac & -ac & c^2 + ac \\ a^2 + ab & b^2 + ab & -ab \end{vmatrix} = (ab + bc + ca)^3$

37. Question

Prove the following identities –

$$\begin{vmatrix} x + \lambda & 2x & 2x \\ 2x & x + \lambda & 2x \\ 2x & 2x & x + \lambda \end{vmatrix} = (5x + \lambda)(\lambda - x)^2$$

Answer

Let $\Delta = \begin{vmatrix} x+\lambda & 2x & 2x \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix}$

Recall that the value of a determinant remains same if we apply the operation $R_i \rightarrow R_i + kR_j$ or $C_i \rightarrow C_i + kC_j$.

Applying $R_1 \rightarrow R_1 + R_2$, we get

$$\Delta = \begin{vmatrix} x+\lambda+2x & 2x+(x+\lambda) & 2x+2x \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 3x+\lambda & 3x+\lambda & 4x \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 + R_3$, we get

$$\Delta = \begin{vmatrix} 3x+\lambda+2x & 3x+\lambda+2x & 4x+(x+\lambda) \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 5x+\lambda & 5x+\lambda & 5x+\lambda \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix}$$

Taking the term $(5x + \lambda)$ common from R_1, we get

$$\Delta = (5x+\lambda)\begin{vmatrix} 1 & 1 & 1 \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 - C_1$, we get

$$\Delta = (5x+\lambda)\begin{vmatrix} 1 & 1-1 & 1 \\ 2x & x+\lambda-2x & 2x \\ 2x & 2x-2x & x+\lambda \end{vmatrix}$$

$$\Rightarrow \Delta = (5x+\lambda)\begin{vmatrix} 1 & 0 & 1 \\ 2x & \lambda-x & 2x \\ 2x & 0 & x+\lambda \end{vmatrix}$$

Applying $C_3 \rightarrow C_3 - C_1$, we get

$$\Delta = (5x+\lambda)\begin{vmatrix} 1 & 0 & 1-1 \\ 2x & \lambda-x & 2x-2x \\ 2x & 0 & x+\lambda-2x \end{vmatrix}$$

$$\Rightarrow \Delta = (5x+\lambda)\begin{vmatrix} 1 & 0 & 0 \\ 2x & \lambda-x & 0 \\ 2x & 0 & \lambda-x \end{vmatrix}$$

Expanding the determinant along R_1, we have

$\Delta = (5x + \lambda)[(1)(\lambda - x)(\lambda - x)]$

$\therefore \Delta = (5x + \lambda)(\lambda - x)^2$

Thus, $\begin{vmatrix} x+\lambda & 2x & 2x \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix} = (5x+\lambda)(\lambda-x)^2$

38. Question

Prove the following identities –

$$\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$$

Answer

Let $\Delta = \begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$

Recall that the value of a determinant remains same if we apply the operation $R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$.

Applying $R_1 \to R_1 + R_2$, we get

$$\Delta = \begin{vmatrix} x+4+2x & 2x+(x+4) & 2x+2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 3x+4 & 3x+4 & 4x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

Applying $R_1 \to R_1 + R_3$, we get

$$\Delta = \begin{vmatrix} 3x+4+2x & 3x+4+2x & 4x+(x+4) \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 5x+4 & 5x+4 & 5x+4 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

Taking the term (5x + 4) common from R_1, we get

$$\Delta = (5x+4)\begin{vmatrix} 1 & 1 & 1 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$, we get

$$\Delta = (5x+4)\begin{vmatrix} 1 & 1-1 & 1 \\ 2x & x+4-2x & 2x \\ 2x & 2x-2x & x+4 \end{vmatrix}$$

$$\Rightarrow \Delta = (5x+4)\begin{vmatrix} 1 & 0 & 1 \\ 2x & 4-x & 2x \\ 2x & 0 & x+4 \end{vmatrix}$$

Applying $C_3 \to C_3 - C_1$, we get

$$\Delta = (5x+4)\begin{vmatrix} 1 & 0 & 1-1 \\ 2x & 4-x & 2x-2x \\ 2x & 0 & x+4-2x \end{vmatrix}$$

$$\Rightarrow \Delta = (5x+4)\begin{vmatrix} 1 & 0 & 0 \\ 2x & 4-x & 0 \\ 2x & 0 & 4-x \end{vmatrix}$$

Expanding the determinant along R_1, we have

$\Delta = (5x + 4)[(1)(4 - x)(4 - x)]$

$\therefore \Delta = (5x + 4)(4 - x)^2$

Thus, $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$

39. Question

Prove the following identities –

$$\begin{vmatrix} y+z & z & y \\ z & z+x & x \\ y & x & x+y \end{vmatrix} = 4xyz$$

Answer

Let $\Delta = \begin{vmatrix} y+z & z & y \\ z & z+x & x \\ y & x & x+y \end{vmatrix}$

Recall that the value of a determinant remains same if we apply the operation $R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$.

Applying $R_1 \to R_1 - R_2$, we get

$$\Delta = \begin{vmatrix} y+z-z & z-(z+x) & y-x \\ z & z+x & x \\ y & x & x+y \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} y & -x & y-x \\ z & z+x & x \\ y & x & x+y \end{vmatrix}$$

Applying $R_1 \to R_1 - R_3$, we get

$$\Delta = \begin{vmatrix} y-y & -x-x & y-x-(x+y) \\ z & z+x & x \\ y & x & x+y \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 0 & -2x & -2x \\ z & z+x & x \\ y & x & x+y \end{vmatrix}$$

Taking the term (–2x) common from R_1, we get

$$\Delta = (-2x)\begin{vmatrix} 0 & 1 & 1 \\ z & z+x & x \\ y & x & x+y \end{vmatrix}$$

Applying $C_2 \to C_2 - C_3$, we get

$$\Delta = (-2x)\begin{vmatrix} 0 & 1-1 & 1 \\ z & z+x-x & x \\ y & x-(x+y) & x+y \end{vmatrix}$$

$$\Rightarrow \Delta = (-2x)\begin{vmatrix} 0 & 0 & 1 \\ z & z & x \\ y & -y & x+y \end{vmatrix}$$

Expanding the determinant along R_1, we have

$\Delta = (-2x)[(z)(-y) - (y)(z)]$

$\Rightarrow \Delta = (-2x)(-yz - yz)$

$\Rightarrow \Delta = (-2x)(-2yz)$

$\therefore \Delta = 4xyz$

Thus, $\begin{vmatrix} y+z & z & y \\ z & z+x & x \\ y & x & x+y \end{vmatrix} = 4xyz$

40. Question

Prove the following identities –

$$\begin{vmatrix} -a(b^2+c^2-a^2) & 2b^3 & 2c^3 \\ 2a^3 & -b(c^2+a^2-b^2) & 2c^3 \\ 2a^3 & 2b^3 & -c(a^2+b^2-c^2) \end{vmatrix} = abc(a^2+b^2+c^2)^3$$

Answer

Let $\Delta = \begin{vmatrix} -a(b^2+c^2-a^2) & 2b^3 & 2c^3 \\ 2a^3 & -b(c^2+a^2-b^2) & 2c^3 \\ 2a^3 & 2b^3 & -c(a^2+b^2-c^2) \end{vmatrix}$

Taking a, b and c common from C_1, C_2 and C_3, we get

$$\Delta = (abc)\begin{vmatrix} -(b^2+c^2-a^2) & 2b^2 & 2c^2 \\ 2a^2 & -(c^2+a^2-b^2) & 2c^2 \\ 2a^2 & 2b^2 & -(a^2+b^2-c^2) \end{vmatrix}$$

Recall that the value of a determinant remains same if we apply the operation $R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$.

Applying $R_1 \to R_1 - R_2$, we get

$$\Delta = (abc)\begin{vmatrix} -(b^2+c^2-a^2)-2a^2 & 2b^2-[-(c^2+a^2-b^2)] & 2c^2-2c^2 \\ 2a^2 & -(c^2+a^2-b^2) & 2c^2 \\ 2a^2 & 2b^2 & -(a^2+b^2-c^2) \end{vmatrix}$$

$$\Rightarrow \Delta = (abc)\begin{vmatrix} -(a^2+b^2+c^2) & a^2+b^2+c^2 & 0 \\ 2a^2 & -(c^2+a^2-b^2) & 2c^2 \\ 2a^2 & 2b^2 & -(a^2+b^2-c^2) \end{vmatrix}$$

Taking the term $(a^2 + b^2 + c^2)$ common from R_1, we get

$$\Delta = (abc)(a^2+b^2+c^2)\begin{vmatrix} -1 & 1 & 0 \\ 2a^2 & -(c^2+a^2-b^2) & 2c^2 \\ 2a^2 & 2b^2 & -(a^2+b^2-c^2) \end{vmatrix}$$

Applying $R_2 \to R_2 - R_3$, we get

$$\Delta = (abc)(a^2+b^2+c^2)\begin{vmatrix} -1 & 1 & 0 \\ 2a^2-2a^2 & -(c^2+a^2-b^2)-2b^2 & 2c^2-[-(a^2+b^2-c^2)] \\ 2a^2 & 2b^2 & -(a^2+b^2-c^2) \end{vmatrix}$$

$$\Rightarrow \Delta = (abc)(a^2+b^2+c^2)\begin{vmatrix} -1 & 1 & 0 \\ 0 & -(a^2+b^2+c^2) & (a^2+b^2+c^2) \\ 2a^2 & 2b^2 & -(a^2+b^2-c^2) \end{vmatrix}$$

Taking the term $(a^2 + b^2 + c^2)$ common from R_2, we get

$$\Delta = (abc)(a^2+b^2+c^2)^2\begin{vmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2a^2 & 2b^2 & -(a^2+b^2-c^2) \end{vmatrix}$$

Applying $C_2 \to C_2 + C_1$, we get

$$\Delta = (abc)(a^2+b^2+c^2)^2\begin{vmatrix} -1 & 1+(-1) & 0 \\ 0 & -1+0 & 1 \\ 2a^2 & 2b^2+2a^2 & -(a^2+b^2-c^2) \end{vmatrix}$$

$$\Rightarrow \Delta = (abc)(a^2+b^2+c^2)^2 \begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 1 \\ 2a^2 & 2b^2+2a^2 & -(a^2+b^2-c^2) \end{vmatrix}$$

Expanding the determinant along R_1, we have

$\Delta = (abc)(a^2 + b^2 + c^2)^2(-1)[(a^2 + b^2 - c^2) - (2b^2 + 2a^2)]$

$\Rightarrow \Delta = (abc)(a^2 + b^2 + c^2)^2[-(a^2 + b^2 - c^2) + (2b^2 + 2a^2)]$

$\Rightarrow \Delta = (abc)(a^2 + b^2 + c^2)^2[-a^2 - b^2 + c^2 + 2b^2 + 2a^2]$

$\Rightarrow \Delta = (abc)(a^2 + b^2 + c^2)^2[a^2 + b^2 + c^2]$

$\therefore \Delta = (abc)(a^2 + b^2 + c^2)^3$

Thus, $\begin{vmatrix} -a(b^2+c^2-a^2) & 2b^3 & 2c^3 \\ 2a^3 & -b(c^2+a^2-b^2) & 2c^3 \\ 2a^3 & 2b^3 & -c(a^2+b^2-c^2) \end{vmatrix} = abc(a^2+b^2+c^2)^3$

41. Question

Prove the following identities –

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+a \end{vmatrix} = a^3+3a^2$$

Answer

Let $\Delta = \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+a \end{vmatrix}$

Recall that the value of a determinant remains same if we apply the operation $R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$.

Applying $R_1 \to R_1 + R_2$, we get

$$\Delta = \begin{vmatrix} 1+a+1 & 1+(1+a) & 1+1 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+a \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 2+a & 2+a & 2 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+a \end{vmatrix}$$

Applying $R_1 \to R_1 + R_3$, we get

$$\Delta = \begin{vmatrix} 2+a+1 & 2+a+1 & 2+(1+a) \\ 1 & 1+a & 1 \\ 1 & 1 & 1+a \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 3+a & 3+a & 3+a \\ 1 & 1+a & 1 \\ 1 & 1 & 1+a \end{vmatrix}$$

Taking the term $(3 + a)$ common from R_1, we get

$$\Delta = (3+a)\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+a \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$, we get

$$\Delta=(3+a)\begin{vmatrix}1 & 1-1 & 1\\ 1 & 1+a-1 & 1\\ 1 & 1-1 & 1+a\end{vmatrix}$$

$$\Rightarrow \Delta=(3+a)\begin{vmatrix}1 & 0 & 1\\ 1 & a & 1\\ 1 & 0 & 1+a\end{vmatrix}$$

Applying $C_3 \rightarrow C_3 - C_1$, we get

$$\Delta=(3+a)\begin{vmatrix}1 & 0 & 1-1\\ 1 & a & 1-1\\ 1 & 0 & 1+a-1\end{vmatrix}$$

$$\Rightarrow \Delta=(3+a)\begin{vmatrix}1 & 0 & 0\\ 1 & a & 0\\ 1 & 0 & a\end{vmatrix}$$

Expanding the determinant along R_1, we have

$\Delta = (3 + a)(1)[(a)(a) - 0]$

$\Rightarrow \Delta = (3 + a)(a^2)$

$\therefore \Delta = a^3 + 3a^2$

Thus, $\begin{vmatrix}1+a & 1 & 1\\ 1 & 1+a & 1\\ 1 & 1 & 1+a\end{vmatrix}=a^3+3a^2$

42. Question

Prove the following identities –

$$\begin{vmatrix}2y & y-z-x & 2y\\ 2z & 2z & z-x-y\\ x-y-z & 2x & 2x\end{vmatrix}=(x+y+z)^3$$

Answer

Let $\Delta=\begin{vmatrix}2y & y-z-x & 2y\\ 2z & 2z & z-x-y\\ x-y-z & 2x & 2x\end{vmatrix}$

Recall that the value of a determinant remains same if we apply the operation $R_i \rightarrow R_i + kR_j$ or $C_i \rightarrow C_i + kC_j$.

Applying $R_1 \rightarrow R_1 + R_2$, we get

$$\Delta=\begin{vmatrix}2y+2z & y-z-x+2z & 2y+(z-x-y)\\ 2z & 2z & z-x-y\\ x-y-z & 2x & 2x\end{vmatrix}$$

$$\Rightarrow \Delta=\begin{vmatrix}2y+2z & y+z-x & z-x+y\\ 2z & 2z & z-x-y\\ x-y-z & 2x & 2x\end{vmatrix}$$

Applying $R_1 \rightarrow R_1 + R_3$, we get

$$\Delta=\begin{vmatrix}2y+2z+(x-y-z) & y+z-x+2x & z-x+y+2x\\ 2z & 2z & z-x-y\\ x-y-z & 2x & 2x\end{vmatrix}$$

$$\Rightarrow \Delta=\begin{vmatrix}x+y+z & x+y+z & x+y+z\\ 2z & 2z & z-x-y\\ x-y-z & 2x & 2x\end{vmatrix}$$

Taking the term (x + y + z) common from R_1, we get

$$\Delta=(x+y+z)\begin{vmatrix}1 & 1 & 1\\ 2z & 2z & z-x-y\\ x-y-z & 2x & 2x\end{vmatrix}$$

Applying $C_2 \rightarrow C_2 - C_1$, we get

$$\Delta=(x+y+z)\begin{vmatrix}1 & 1-1 & 1\\ 2z & 2z-2z & z-x-y\\ x-y-z & 2x-(x-y-z) & 2x\end{vmatrix}$$

$$\Rightarrow \Delta=(x+y+z)\begin{vmatrix}1 & 0 & 1\\ 2z & 0 & z-x-y\\ x-y-z & x+y+z & 2x\end{vmatrix}$$

Applying $C_3 \rightarrow C_3 - C_1$, we get

$$\Delta=(x+y+z)\begin{vmatrix}1 & 0 & 1-1\\ 2z & 0 & z-x-y-2z\\ x-y-z & x+y+z & 2x-(x-y-z)\end{vmatrix}$$

$$\Rightarrow \Delta=(x+y+z)\begin{vmatrix}1 & 0 & 0\\ 2z & 0 & -(x+y+z)\\ x-y-z & x+y+z & x+y+z\end{vmatrix}$$

Expanding the determinant along R_1, we have

$\Delta = (x + y + z)(1)[0 - (-(x + y + z)(x + y + z))]$

$\Rightarrow \Delta = (x + y + z)(x + y + z)(x + y + z)$

$\therefore \Delta = (x + y + z)^3$

Thus, $\begin{vmatrix}2y & y-z-x & 2y\\ 2z & 2z & z-x-y\\ x-y-z & 2x & 2x\end{vmatrix}=(x+y+z)^3$

43. Question

Prove the following identities –

$$\begin{vmatrix}y+z & x & y\\ z+x & z & x\\ x+y & y & z\end{vmatrix}=(z+y+z)(x-z)^2$$

Answer

Let $\Delta=\begin{vmatrix}y+z & x & y\\ z+x & z & x\\ x+y & y & z\end{vmatrix}$

Recall that the value of a determinant remains same if we apply the operation $R_i \rightarrow R_i + kR_j$ or $C_i \rightarrow C_i + kC_j$.

Applying $R_1 \rightarrow R_1 + R_2$, we get

$$\Delta=\begin{vmatrix}y+z+(z+x) & x+z & y+x\\ z+x & z & x\\ x+y & y & z\end{vmatrix}$$

$$\Rightarrow \Delta=\begin{vmatrix}x+y+2z & x+z & y+x\\ z+x & z & x\\ x+y & y & z\end{vmatrix}$$

Applying $R_1 \rightarrow R_1 + R_3$, we get

$$\Delta = \begin{vmatrix} x+y+2z+(x+y) & x+z+y & y+x+z \\ z+x & z & x \\ x+y & y & z \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 2(x+y+z) & x+y+z & x+y+z \\ z+x & z & x \\ x+y & y & z \end{vmatrix}$$

Taking the term (x + y + z) common from R_1, we get

$$\Delta = (x+y+z)\begin{vmatrix} 2 & 1 & 1 \\ z+x & z & x \\ x+y & y & z \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 - C_2$, we get

$$\Delta = (x+y+z)\begin{vmatrix} 2-1 & 1 & 1 \\ z+x-z & z & x \\ x+y-y & y & z \end{vmatrix}$$

$$\Rightarrow \Delta = (x+y+z)\begin{vmatrix} 1 & 1 & 1 \\ x & z & x \\ x & y & z \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 - C_3$, we get

$$\Delta = (x+y+z)\begin{vmatrix} 1-1 & 1 & 1 \\ x-x & z & x \\ x-z & y & z \end{vmatrix}$$

$$\Rightarrow \Delta = (x+y+z)\begin{vmatrix} 0 & 1 & 1 \\ 0 & z & x \\ x-z & y & z \end{vmatrix}$$

Expanding the determinant along C_1, we have

$\Delta = (x + y + z)(x - z)[(1)(x) - (z)(1)]$

$\Rightarrow \Delta = (x + y + z)(x - z)(x - z)$

$\therefore \Delta = (x + y + z)(x - z)^2$

Thus, $\begin{vmatrix} y+z & x & y \\ z+x & z & x \\ x+y & y & z \end{vmatrix} = (x+y+z)(x-z)^2$

44. Question

Prove the following identities –

$$\begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix} = a^2(a+x+y+z)$$

Answer

Let $\Delta = \begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix}$

Recall that the value of a determinant remains same if we apply the operation $R_i \rightarrow R_i + kR_j$ or $C_i \rightarrow C_i + kC_j$.

Applying $C_1 \rightarrow C_1 + C_2$, we get

$$\Delta = \begin{vmatrix} a+x+y & y & z \\ x+(a+y) & a+y & z \\ x+y & y & a+z \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} a+x+y & y & z \\ a+x+y & a+y & z \\ x+y & y & a+z \end{vmatrix}$$

Applying $C_1 \to C_1 + C_3$, we get

$$\Delta = \begin{vmatrix} a+x+y+z & y & z \\ a+x+y+z & a+y & z \\ x+y+(a+z) & y & a+z \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} a+x+y+z & y & z \\ a+x+y+z & a+y & z \\ a+x+y+z & y & a+z \end{vmatrix}$$

Taking the term (a + x + y + z) common from C_1, we get

$$\Delta = (a+x+y+z)\begin{vmatrix} 1 & y & z \\ 1 & a+y & z \\ 1 & y & a+z \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$, we get

$$\Delta = (a+x+y+z)\begin{vmatrix} 1 & y & z \\ 1-1 & a+y-y & z-z \\ 1 & y & a+z \end{vmatrix}$$

$$\Rightarrow \Delta = (a+x+y+z)\begin{vmatrix} 1 & y & z \\ 0 & a & 0 \\ 1 & y & a+z \end{vmatrix}$$

Applying $R_3 \to R_3 - R_1$, we get

$$\Delta = (a+x+y+z)\begin{vmatrix} 1 & y & z \\ 0 & a & 0 \\ 1-1 & y-y & a+z-z \end{vmatrix}$$

$$\Rightarrow \Delta = (a+x+y+z)\begin{vmatrix} 1 & y & z \\ 0 & a & 0 \\ 0 & 0 & a \end{vmatrix}$$

Expanding the determinant along C_1, we have

Δ = (a + x + y + z)(1)[(a)(a) – (0)(0)]

$\Rightarrow \Delta$ = (a + x + y + z)(a)(a)

$\therefore \Delta = a^2(a + x + y + z)$

Thus, $\begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix} = a^2(a+x+y+z)$

45. Question

Prove the following identities –

$\begin{vmatrix} a^3 & 2 & a \\ b^3 & 2 & b \\ c^3 & 2 & c \end{vmatrix}$ = 2(a - b)(b - c)(c - a)(a + b + c)

Answer

Let $\Delta = \begin{vmatrix} a^3 & 2 & a \\ b^3 & 2 & b \\ c^3 & 2 & c \end{vmatrix}$

Taking 2 common from C_2, we get

$$\Delta = 2\begin{vmatrix} a^3 & 1 & a \\ b^3 & 1 & b \\ c^3 & 1 & c \end{vmatrix}$$

Recall that the value of a determinant remains same if we apply the operation $R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$.

Applying $R_2 \to R_2 - R_1$, we get

$$\Delta = 2\begin{vmatrix} a^3 & 1 & a \\ b^3 - a^3 & 1 - 1 & b - a \\ c^3 & 1 & c \end{vmatrix}$$

$$\Rightarrow \Delta = 2\begin{vmatrix} a^3 & 1 & a \\ b^3 - a^3 & 0 & b - a \\ c^3 & 1 & c \end{vmatrix}$$

Applying $R_3 \to R_3 - R_1$, we get

$$\Delta = 2\begin{vmatrix} a^3 & 1 & a \\ b^3 - a^3 & 0 & b - a \\ c^3 - a^3 & 1 - 1 & c - a \end{vmatrix}$$

$$\Rightarrow \Delta = 2\begin{vmatrix} a^3 & 1 & a \\ b^3 - a^3 & 0 & b - a \\ c^3 - a^3 & 0 & c - a \end{vmatrix}$$

We have the identity $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$$\Rightarrow \Delta = 2\begin{vmatrix} a^3 & 1 & a \\ (b-a)(b^2 + ba + a^2) & 0 & b - a \\ (c-a)(c^2 + ca + a^2) & 0 & c - a \end{vmatrix}$$

Taking (b – a) and (c – a) common from R_2 and R_3, we get

$$\Delta = 2(b-a)(c-a)\begin{vmatrix} a^3 & 1 & a \\ b^2 + ba + a^2 & 0 & 1 \\ c^2 + ca + a^2 & 0 & 1 \end{vmatrix}$$

We know that the sign of a determinant changes if any two rows or columns are interchanged.

By interchanging C_1 and C_2, we get

$$\Delta = -2(b-a)(c-a)\begin{vmatrix} 1 & a^3 & a \\ 0 & b^2 + ba + a^2 & 1 \\ 0 & c^2 + ca + a^2 & 1 \end{vmatrix}$$

Expanding the determinant along C_1, we have

$\Delta = -2(b - a)(c - a)(1)[(b^2 + ba + a^2) - (c^2 + ca + a^2)]$

$\Rightarrow \Delta = 2(a - b)(c - a)[b^2 + ba + a^2 - c^2 - ca - a^2]$

$\Rightarrow \Delta = 2(a - b)(c - a)[b^2 + ba - c^2 - ca]$

$\Rightarrow \Delta = 2(a - b)(c - a)[b^2 - c^2 + (ba - ca)]$

$\Rightarrow \Delta = 2(a - b)(c - a)[(b - c)(b + c) + (b - c)a]$

$\Rightarrow \Delta = 2(a - b)(c - a)(b - c)(b + c + a)$

$\therefore \Delta = 2(a - b)(b - c)(c - a)(a + b + c)$

Thus, $\begin{vmatrix} a^3 & 2 & a \\ b^3 & 2 & b \\ c^3 & 2 & c \end{vmatrix} = 2(a-b)(b-c)(c-a)(a+b+c)$

46. Question

Without expanding, prove that $\begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix} = \begin{vmatrix} x & y & z \\ p & q & r \\ a & b & c \end{vmatrix} = \begin{vmatrix} y & b & q \\ x & a & p \\ z & c & r \end{vmatrix}$.

Answer

Let $\Delta = \begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix}$

We know that the sign of a determinant changes if any two rows or columns are interchanged.

By interchanging R_1 and R_2, we get

$$\Delta = -\begin{vmatrix} x & y & z \\ a & b & c \\ p & q & r \end{vmatrix}$$

By interchanging R_2 and R_3, we get

$$\Delta = -\left(-\begin{vmatrix} x & y & z \\ p & q & r \\ a & b & c \end{vmatrix}\right)$$

$$\Rightarrow \Delta = \begin{vmatrix} x & y & z \\ p & q & r \\ a & b & c \end{vmatrix}$$

Hence, $\begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix} = \begin{vmatrix} x & y & z \\ p & q & r \\ a & b & c \end{vmatrix}$

Let us once again consider $\Delta = \begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix}$

By interchanging R_1 and R_2, we get

$$\Delta = -\begin{vmatrix} x & y & z \\ a & b & c \\ p & q & r \end{vmatrix}$$

By interchanging C_1 and C_2, we get

$$\Delta = -\left(-\begin{vmatrix} y & x & z \\ b & a & c \\ q & p & r \end{vmatrix}\right)$$

$$\Rightarrow \Delta = \begin{vmatrix} y & x & z \\ b & a & c \\ q & p & r \end{vmatrix}$$

Recall that the value of a determinant remains same if it its rows and columns are interchanged.

$$\Rightarrow \Delta = \begin{vmatrix} y & b & q \\ x & a & p \\ z & c & r \end{vmatrix}$$

Hence, $\begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix} = \begin{vmatrix} y & b & q \\ x & a & p \\ z & c & r \end{vmatrix}$

Thus, $\begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix} = \begin{vmatrix} x & y & z \\ p & q & r \\ a & b & c \end{vmatrix} = \begin{vmatrix} y & b & q \\ x & a & p \\ z & c & r \end{vmatrix}$

47. Question

Show that $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$ where a,b,c are in A.P.

Answer

Let $\Delta = \begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$

Recall that the value of a determinant remains same if we apply the operation $R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$.

Applying $R_1 \to R_1 + R_3$, we get

$$\Delta = \begin{vmatrix} x+1+(x+3) & x+2+(x+4) & x+a+(x+c) \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 2x+4 & 2x+6 & 2x+(a+c) \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$$

Given that a, b and c are in an A.P. Using the definition of an arithmetic progression, we have

b – a = c – b

⇒ b + b = c + a

⇒ 2b = c + a

∴ a + c = 2b

By substituting this in the above equation to find Δ, we get

$$\Delta = \begin{vmatrix} 2x+4 & 2x+6 & 2x+2b \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 2(x+2) & 2(x+3) & 2(x+b) \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$$

Taking 2 common from R_1, we get

$$\Delta = 2\begin{vmatrix} x+2 & x+3 & x+b \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$, we get

$$\Delta = 2\begin{vmatrix} x+2-(x+2) & x+3-(x+3) & x+b-(x+b) \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$$

$$\Rightarrow \Delta = 2\begin{vmatrix} 0 & 0 & 0 \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$$

∴ Δ = 0

Thus, $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$ when a, b and c are in A.P.

48. Question

Show that $\begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ x-1 & x-2 & x-\gamma \end{vmatrix} = 0$ where α, β and γ are in A.P.

Answer

Let $\Delta = \begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ x-1 & x-2 & x-\gamma \end{vmatrix}$

Recall that the value of a determinant remains same if we apply the operation $R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$.

Applying $R_1 \to R_1 + R_3$, we get

$$\Delta = \begin{vmatrix} x-3+(x-1) & x-4+(x-2) & x-\alpha+(x-\gamma) \\ x-2 & x-3 & x-\beta \\ x-1 & x-2 & x-\gamma \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 2x-4 & 2x-6 & 2x-(\alpha+\gamma) \\ x-2 & x-3 & x-\beta \\ x-1 & x-2 & x-\gamma \end{vmatrix}$$

Given that α, β and γ are in an A.P. Using the definition of an arithmetic progression, we have

β - α = γ - β

⇒ β + β = γ + α

⇒ 2β = γ + α

∴ α + γ = 2β

By substituting this in the above equation to find Δ, we get

$$\Delta = \begin{vmatrix} 2x-4 & 2x-6 & 2x-2\beta \\ x-2 & x-3 & x-\beta \\ x-1 & x-2 & x-\gamma \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 2(x-2) & 2(x-3) & 2(x-\beta) \\ x-2 & x-3 & x-\beta \\ x-1 & x-2 & x-\gamma \end{vmatrix}$$

Taking 2 common from R_1, we get

$$\Delta = 2\begin{vmatrix} x-2 & x-3 & x-\beta \\ x-2 & x-3 & x-\beta \\ x-1 & x-2 & x-\gamma \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$, we get

$$\Delta = 2\begin{vmatrix} x-2-(x-2) & x-3-(x-3) & x-\beta-(x-\beta) \\ x-2 & x-3 & x-\beta \\ x-1 & x-2 & x-\gamma \end{vmatrix}$$

$$\Rightarrow \Delta = 2\begin{vmatrix} 0 & 0 & 0 \\ x-2 & x-3 & x-\beta \\ x-1 & x-2 & x-\gamma \end{vmatrix}$$

∴ Δ = 0

Thus, $\begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ x-1 & x-2 & x-\gamma \end{vmatrix} = 0$ when α, β and γ are in A.P.

49. Question

If a, b, c are real numbers such that $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$, then show that either a + b + c = 0 or a = b = c.

Answer

Let $\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$

Given that Δ = 0.

Recall that the value of a determinant remains same if we apply the operation $R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$.

Applying $R_1 \to R_1 + R_2$, we get

$$\Delta = \begin{vmatrix} b+c+(c+a) & c+a+(a+b) & a+b+(b+c) \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} a+b+2c & 2a+b+c & a+2b+c \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

Applying $R_1 \to R_1 + R_3$, we get

$$\Delta = \begin{vmatrix} a+b+2c+(a+b) & 2a+b+c+(b+c) & a+2b+c+(c+a) \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 2(a+b+c) & 2(a+b+c) & 2(a+2b+c) \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

Taking the term 2(a + b + c) common from R_1, we get

$$\Delta = 2(a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$, we get

$$\Delta = 2(a+b+c)\begin{vmatrix} 1 & 1-1 & 1 \\ c+a & a+b-(c+a) & b+c \\ a+b & b+c-(a+b) & c+a \end{vmatrix}$$

$$\Rightarrow \Delta = 2(a+b+c)\begin{vmatrix} 1 & 0 & 1 \\ c+a & b-c & b+c \\ a+b & c-a & c+a \end{vmatrix}$$

Applying $C_3 \to C_3 - C_1$, we get

$$\Delta = 2(a+b+c)\begin{vmatrix} 1 & 0 & 1-1 \\ c+a & b-c & b+c-(c+a) \\ a+b & c-a & c+a-(a+b) \end{vmatrix}$$

$$\Rightarrow \Delta = 2(a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ c+a & b-c & b-a \\ a+b & c-a & c-b \end{vmatrix}$$

Expanding the determinant along R_1, we have

$\Delta = 2(a + b + c)(1)[(b - c)(c - b) - (c - a)(b - a)]$

$\Rightarrow \Delta = 2(a + b + c)(bc - b^2 - c^2 + cb - cb + ca + ab - a^2)$

$\therefore \Delta = 2(a + b + c)(ab + bc + ca - a^2 - b^2 - c^2)$

We have $\Delta = 0$

$\Rightarrow 2(a + b + c)(ab + bc + ca - a^2 - b^2 - c^2) = 0$

$\Rightarrow (a + b + c)(ab + bc + ca - a^2 - b^2 - c^2) = 0$

Case – I:

$a + b + c = 0$

Case – II:

$ab + bc + ca - a^2 - b^2 - c^2 = 0$

$\Rightarrow a^2 + b^2 + c^2 - ab - bc - ca = 0$

Multiplying 2 on both sides, we have

$2(a^2 + b^2 + c^2 - ab - bc - ca) = 0$

$\Rightarrow 2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca = 0$

$\Rightarrow a^2 - 2ab + b^2 + b^2 - 2bc + c^2 + c^2 - 2ca + a^2 = 0$

$\Rightarrow (a - b)^2 + (b - c)^2 + (c - a)^2 = 0$

We know $(a - b)^2 \geq 0, (b - c)^2 \geq 0, (c - a)^2 \geq 0$

If the sum of three non-negative numbers is zero, then each of the numbers is zero.

$\Rightarrow (a - b)^2 = 0 = (b - c)^2 = (c - a)^2$

$\Rightarrow a - b = 0 = b - c = c - a$

$\Rightarrow a = b = c$

Thus, if $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$, then either $a + b + c = 0$ or $a = b = c$.

50. Question

If $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$, find the value of $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c}, p \neq a, q = b, r \neq c$.

Answer

Let $\Delta = \begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix}$

Given that $\Delta = 0$.

Recall that the value of a determinant remains same if we apply the operation $R_i \rightarrow R_i + kR_j$ or $C_i \rightarrow C_i + kC_j$.

Applying $R_1 \rightarrow R_1 - R_2$, we get

$$\Delta = \begin{vmatrix} p-a & b-q & c-c \\ a & q & c \\ a & b & r \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} p-a & b-q & 0 \\ a & q & c \\ a & b & r \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_3$, we get

$$\Delta = \begin{vmatrix} p-a & b-q & 0 \\ a-a & q-b & c-r \\ a & b & r \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} p-a & b-q & 0 \\ 0 & q-b & c-r \\ a & b & r \end{vmatrix}$$

Expanding the determinant along R_1, we have

$\Delta = (p - a)[(q - b)(r) - (b)(c - r)] - (b - q)[-a(c - r)]$

$\Rightarrow \Delta = r(p - a)(q - b) - b(p - a)(c - r) + a(b - q)(c - r)$

$\therefore \Delta = r(p - a)(q - b) + b(p - a)(r - c) + a(q - b)(r - c)$

We have $\Delta = 0$

$\Rightarrow r(p - a)(q - b) + b(p - a)(r - c) + a(q - b)(r - c) = 0$

On dividing the equation with $(p - a)(q - b)(r - c)$, we get

$$\frac{r(p-a)(q-b)+b(p-a)(r-c)+a(q-b)(r-c)}{(p-a)(q-b)(r-c)} = 0$$

$$\Rightarrow \frac{r}{r-c} + \frac{b}{q-b} + \frac{a}{p-a} = 0$$

$$\Rightarrow \frac{r}{r-c} + \frac{b-q+q}{q-b} + \frac{a-p+p}{p-a} = 0$$

$$\Rightarrow \frac{r}{r-c} + \frac{b-q}{q-b} + \frac{q}{q-b} + \frac{a-p}{p-a} + \frac{p}{p-a} = 0$$

$$\Rightarrow \frac{r}{r-c} + (-1) + \frac{q}{q-b} + (-1) + \frac{p}{p-a} = 0$$

$$\Rightarrow \frac{r}{r-c} + (-1) + \frac{q}{q-b} + (-1) + \frac{p}{p-a} = 0$$

$$\Rightarrow \frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} - 2 = 0$$

$$\therefore \frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$$

Thus, $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$

51. Question

Show that x = 2 is a root of the equation $\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$ and solve it completely.

Answer

Let $\Delta = \begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix}$

We need to find the roots of Δ = 0.

Recall that the value of a determinant remains same if we apply the operation $R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$.

Applying $R_2 \to R_2 - R_1$, we get

$$\Delta = \begin{vmatrix} x & -6 & -1 \\ 2-x & -3x-(-6) & x-3-(-1) \\ -3 & 2x & x+2 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} x & -6 & -1 \\ 2-x & -3x+6 & x-2 \\ -3 & 2x & x+2 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} x & -6 & -1 \\ -(x-2) & -3(x-2) & x-2 \\ -3 & 2x & x+2 \end{vmatrix}$$

Taking the term (x – 2) common from R_2, we get

$$\Delta = (x-2)\begin{vmatrix} x & -6 & -1 \\ -1 & -3 & 1 \\ -3 & 2x & x+2 \end{vmatrix}$$

Applying $R_3 \to R_3 - R_1$, we get

$$\Delta = (x-2)\begin{vmatrix} x & -6 & -1 \\ -1 & -3 & 1 \\ -3-x & 2x-(-6) & x+2-(-1) \end{vmatrix}$$

$$\Rightarrow \Delta = (x-2)\begin{vmatrix} x & -6 & -1 \\ -1 & -3 & 1 \\ -x-3 & 2x+6 & x+3 \end{vmatrix}$$

$$\Rightarrow \Delta = (x-2)\begin{vmatrix} x & -6 & -1 \\ -1 & -3 & 1 \\ -(x+3) & 2(x+3) & x+3 \end{vmatrix}$$

Taking the term (x + 3) common from R_3, we get

$$\Delta = (x-2)(x+3)\begin{vmatrix} x & -6 & -1 \\ -1 & -3 & 1 \\ -1 & 2 & 1 \end{vmatrix}$$

Applying $C_1 \to C_1 + C_3$, we get

$$\Delta = (x-2)(x+3)\begin{vmatrix} x+(-1) & -6 & -1 \\ -1+1 & -3 & 1 \\ -1+1 & 2 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = (x-2)(x+3)\begin{vmatrix} x-1 & -6 & -1 \\ 0 & -3 & 1 \\ 0 & 2 & 1 \end{vmatrix}$$

Expanding the determinant along C_1, we have

Δ = (x – 2)(x + 3)(x – 1)[(–3)(1) – (2)(1)]

⇒ Δ = (x – 2)(x + 3)(x – 1)(–5)

∴ Δ = –5(x – 2)(x + 3)(x – 1)

The given equation is Δ = 0.

⇒ –5(x – 2)(x + 3)(x – 1) = 0

$\Rightarrow (x - 2)(x + 3)(x - 1) = 0$

Case – I:

$x - 2 = 0 \Rightarrow x = 2$

Case – II:

$x + 2 = 0 \Rightarrow x = -3$

Case – III:

$x - 1 = 0 \Rightarrow x = 1$

Thus, 2 is a root of the equation $\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$ and its other roots are –3 and 1.

52 A. Question

Solve the following determinant equations:

$$\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = 0$$

Answer

$$\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = 0$$

Let $\Delta = \begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix}$

We need to find the roots of $\Delta = 0$.

Recall that the value of a determinant remains same if we apply the operation $R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$.

Applying $C_1 \to C_1 + C_2$, we get

$$\Delta = \begin{vmatrix} x+a+b & b & c \\ a+(x+b) & x+b & c \\ a+b & b & x+c \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} x+a+b & b & c \\ x+a+b & x+b & c \\ a+b & b & x+c \end{vmatrix}$$

Applying $C_1 \to C_1 + C_3$, we get

$$\Delta = \begin{vmatrix} x+a+b+c & b & c \\ x+a+b+c & x+b & c \\ a+b+(x+c) & b & x+c \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} x+a+b+c & b & c \\ x+a+b+c & x+b & c \\ x+a+b+c & b & x+c \end{vmatrix}$$

Taking the term $(x + a + b + c)$ common from C_1, we get

$$\Delta = (x+a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & x+b & c \\ 1 & b & x+c \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$, we get

$$\Delta = (x+a+b+c)\begin{vmatrix} 1 & b & c \\ 1-1 & x+b-b & c-c \\ 1 & b & x+c \end{vmatrix}$$

$$\Rightarrow \Delta = (x+a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & x & 0 \\ 1 & b & x+c \end{vmatrix}$$

Applying $R_3 \to R_3 - R_1$, we get

$$\Delta = (x+a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & x & 0 \\ 1-1 & b-b & x+c-c \end{vmatrix}$$

$$\Rightarrow \Delta = (x+a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & x & 0 \\ 0 & 0 & x \end{vmatrix}$$

Expanding the determinant along C_1, we have

$\Delta = (x + a + b + c)(1)[(x)(x) - (0)(0)]$

$\Rightarrow \Delta = (x + a + b + c)(x)(x)$

$\therefore \Delta = x^2(x + a + b + c)$

The given equation is $\Delta = 0$.

$\Rightarrow x^2(x + a + b + c) = 0$

Case – I:

$x^2 = 0 \Rightarrow x = 0$

Case – II:

$x + a + b + c = 0 \Rightarrow x = -(a + b + c)$

Thus, 0 and $-(a + b + c)$ are the roots of the given determinant equation.

52 B. Question

Solve the following determinant equations:

$$\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0, a \neq 0$$

Answer

$$\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0, a \neq 0$$

Let $\Delta = \begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix}$

We need to find the roots of $\Delta = 0$.

Recall that the value of a determinant remains same if we apply the operation $R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$.

Applying $C_1 \to C_1 + C_2$, we get

$$\Delta = \begin{vmatrix} x+a+x & x & x \\ x+(x+a) & x+a & x \\ x+x & x & x+a \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 2x+a & x & x \\ 2x+a & x+a & x \\ 2x & x & x+a \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 + C_3$, we get

$$\Delta = \begin{vmatrix} 2x+a+x & x & x \\ 2x+a+x & x+a & x \\ 2x+(x+a) & x & x+a \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 3x+a & x & x \\ 3x+a & x+a & x \\ 3x+a & x & x+a \end{vmatrix}$$

Taking the term (3x + a) common from C_1, we get

$$\Delta = (3x+a)\begin{vmatrix} 1 & x & x \\ 1 & x+a & x \\ 1 & x & x+a \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_1$, we get

$$\Delta = (3x+a)\begin{vmatrix} 1 & x & x \\ 1-1 & x+a-x & x-x \\ 1 & x & x+a \end{vmatrix}$$

$$\Rightarrow \Delta = (3x+a)\begin{vmatrix} 1 & x & x \\ 0 & a & 0 \\ 1 & x & x+a \end{vmatrix}$$

Applying $R_3 \rightarrow R_3 - R_1$, we get

$$\Delta = (3x+a)\begin{vmatrix} 1 & x & x \\ 0 & a & 0 \\ 1-1 & x-x & x+a-x \end{vmatrix}$$

$$\Rightarrow \Delta = (3x+a)\begin{vmatrix} 1 & x & x \\ 0 & a & 0 \\ 0 & 0 & a \end{vmatrix}$$

Expanding the determinant along C_1, we have

$\Delta = (3x + a)(1)[(a)(a) - (0)(0)]$

$\Rightarrow \Delta = (3x + a)(a)(a)$

$\therefore \Delta = a^2(3x + a)$

The given equation is $\Delta = 0$.

$\Rightarrow a^2(3x + a) = 0$

However, $a \neq 0$ according to the given condition.

$\Rightarrow 3x + a = 0$

$\Rightarrow 3x = -a$

$$\therefore x = -\frac{a}{3}$$

Thus, $-\frac{a}{3}$ is the root of the given determinant equation.

52 C. Question

Solve the following determinant equations:

$$\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$$

Answer

$$\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$$

Let $\Delta = \begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix}$

We need to find the roots of Δ = 0.

Recall that the value of a determinant remains same if we apply the operation $R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$.

Applying $C_1 \to C_1 + C_2$, we get

$$\Delta = \begin{vmatrix} 3x-8+3 & 3 & 3 \\ 3+(3x-8) & 3x-8 & 3 \\ 3+3 & 3 & 3x-8 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 3x-5 & 3 & 3 \\ 3x-5 & 3x-8 & 3 \\ 6 & 3 & 3x-8 \end{vmatrix}$$

Applying $C_1 \to C_1 + C_3$, we get

$$\Delta = \begin{vmatrix} 3x-5+3 & 3 & 3 \\ 3x-5+3 & 3x-8 & 3 \\ 6+(3x-8) & 3 & 3x-8 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 3x-2 & 3 & 3 \\ 3x-2 & 3x-8 & 3 \\ 3x-2 & 3 & 3x-8 \end{vmatrix}$$

Taking the term (3x – 2) common from C_1, we get

$$\Delta = (3x-2)\begin{vmatrix} 1 & 3 & 3 \\ 1 & 3x-8 & 3 \\ 1 & 3 & 3x-8 \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$, we get

$$\Delta = (3x-2)\begin{vmatrix} 1 & 3 & 3 \\ 1-1 & 3x-8-3 & 3-3 \\ 1 & 3 & 3x-8 \end{vmatrix}$$

$$\Rightarrow \Delta = (3x-2)\begin{vmatrix} 1 & 3 & 3 \\ 0 & 3x-11 & 0 \\ 1 & 3 & 3x-8 \end{vmatrix}$$

Applying $R_3 \to R_3 - R_1$, we get

$$\Delta = (3x-2)\begin{vmatrix} 1 & 3 & 3 \\ 0 & 3x-11 & 0 \\ 1-1 & 3-3 & 3x-8-3 \end{vmatrix}$$

$$\Rightarrow \Delta = (3x-2)\begin{vmatrix} 1 & 3 & 3 \\ 0 & 3x-11 & 0 \\ 0 & 0 & 3x-11 \end{vmatrix}$$

Expanding the determinant along C_1, we have

Δ = (3x – 2)(1)[(3x – 11)(3x – 11) – (0)(0)]

$\Rightarrow \Delta = (3x - 2)(3x - 11)(3x - 11)$

$\therefore \Delta = (3x - 2)(3x - 11)^2$

The given equation is $\Delta = 0$.

$\Rightarrow (3x - 2)(3x - 11)^2 = 0$

Case – I:

$3x - 2 = 0$

$\Rightarrow 3x = 2$

$\therefore x = \frac{2}{3}$

Case – II:

$(3x - 11)^2 = 0$

$\Rightarrow 3x - 11 = 0$

$\Rightarrow 3x = 11$

$\therefore x = \frac{11}{3}$

Thus, $\frac{2}{3}$ and $\frac{11}{3}$ are the roots of the given determinant equation.

52 D. Question

Solve the following determinant equations:

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & a & a^2 \\ 1 & b & b^2 \end{vmatrix} = 0, a \neq b$$

Answer

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & a & a^2 \\ 1 & b & b^2 \end{vmatrix} = 0, a \neq b$$

Let $\Delta = \begin{vmatrix} 1 & x & x^2 \\ 1 & a & a^2 \\ 1 & b & b^2 \end{vmatrix}$

We need to find the roots of $\Delta = 0$.

Recall that the value of a determinant remains same if we apply the operation $R_i \rightarrow R_i + kR_j$ or $C_i \rightarrow C_i + kC_j$.

Applying $R_2 \rightarrow R_2 - R_1$, we get

$$\Delta = \begin{vmatrix} 1 & x & x^2 \\ 1 - 1 & a - x & a^2 - x^2 \\ 1 & b & b^2 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & x & x^2 \\ 0 & a - x & a^2 - x^2 \\ 1 & b & b^2 \end{vmatrix}$$

Applying $R_3 \rightarrow R_3 - R_1$, we get

$$\Delta = \begin{vmatrix} 1 & x & x^2 \\ 0 & a-x & a^2-x^2 \\ 1-1 & b-x & b^2-x^2 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & x & x^2 \\ 0 & a-x & a^2-x^2 \\ 0 & b-x & b^2-x^2 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & x & x^2 \\ 0 & a-x & (a-x)(a+x) \\ 0 & b-x & (b-x)(b+x) \end{vmatrix}$$

Taking (a – x) and (b – x) common from R_2 and R_3, we get

$$\Delta = (a-x)(b-x)\begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & a+x \\ 0 & 1 & b+x \end{vmatrix}$$

Expanding the determinant along C_1, we have

$\Delta = (a - x)(b - x)(1)[(1)(b + x) - (1)(a + x)]$

$\Rightarrow \Delta = (a - x)(b - x)[b + x - a - x]$

$\therefore \Delta = (a - x)(b - x)(b - a)$

The given equation is $\Delta = 0$.

$\Rightarrow (a - x)(b - x)(b - a) = 0$

However, $a \neq b$ according to the given condition.

$\Rightarrow (a - x)(b - x) = 0$

<u>Case – I</u>:

$a - x = 0 \Rightarrow x = a$

<u>Case – II</u>:

$b - x = 0 \Rightarrow x = b$

Thus, a and b are the roots of the given determinant equation.

52 E. Question

Solve the following determinant equations:

$$\begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix} = 0$$

Answer

$$\begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix} = 0$$

Let $\Delta = \begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix}$

We need to find the roots of $\Delta = 0$.

Recall that the value of a determinant remains same if we apply the operation $R_i \rightarrow R_i + kR_j$ or $C_i \rightarrow C_i + kC_j$.

Applying $C_1 \rightarrow C_1 + C_2$, we get

$$\Delta = \begin{vmatrix} x+1+3 & 3 & 5 \\ 2+(x+2) & x+2 & 5 \\ 2+3 & 3 & x+4 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} x+4 & 3 & 5 \\ x+4 & x+2 & 5 \\ 5 & 3 & x+4 \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 + C_3$, we get

$$\Delta = \begin{vmatrix} x+4+5 & 3 & 5 \\ x+4+5 & x+2 & 5 \\ 5+(x+4) & 3 & x+4 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} x+9 & 3 & 5 \\ x+9 & x+2 & 5 \\ x+9 & 3 & x+4 \end{vmatrix}$$

Taking the term (x + 9) common from C_1, we get

$$\Delta = (x+9)\begin{vmatrix} 1 & 3 & 5 \\ 1 & x+2 & 5 \\ 1 & 3 & x+4 \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_1$, we get

$$\Delta = (x+9)\begin{vmatrix} 1 & 3 & 5 \\ 1-1 & x+2-3 & 5-5 \\ 1 & 3 & x+4 \end{vmatrix}$$

$$\Rightarrow \Delta = (x+9)\begin{vmatrix} 1 & 3 & 5 \\ 0 & x-1 & 0 \\ 1 & 3 & x+4 \end{vmatrix}$$

Applying $R_3 \rightarrow R_3 - R_1$, we get

$$\Delta = (x+9)\begin{vmatrix} 1 & 3 & 5 \\ 0 & x-1 & 0 \\ 1-1 & 3-3 & x+4-5 \end{vmatrix}$$

$$\Rightarrow \Delta = (x+9)\begin{vmatrix} 1 & 3 & 5 \\ 0 & x-1 & 0 \\ 0 & 0 & x-1 \end{vmatrix}$$

Expanding the determinant along C_1, we have

$\Delta = (x + 9)(1)[(x - 1)(x - 1) - (0)(0)]$

$\Rightarrow \Delta = (x + 9)(x - 1)(x - 1)$

$\therefore \Delta = (x + 9)(x - 1)^2$

The given equation is $\Delta = 0$.

$\Rightarrow x^2(x + a + b + c) = 0$

<u>Case – I</u>:

$x + 9 = 0 \Rightarrow x = -9$

<u>Case – II</u>:

$(x - 1)^2 = 0$

$\Rightarrow x - 1 = 0$

$\therefore x = 1$

Thus, –9 and 1 are the roots of the given determinant equation.

52 F. Question

Solve the following determinant equations:

$$\begin{vmatrix} 1 & x & x^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = 0, b \neq c$$

Answer

$$\begin{vmatrix} 1 & x & x^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = 0, b \neq c$$

Let $\Delta = \begin{vmatrix} 1 & x & x^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix}$

We need to find the roots of $\Delta = 0$.

Recall that the value of a determinant remains same if we apply the operation $R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$.

Applying $R_2 \to R_2 - R_1$, we get

$$\Delta = \begin{vmatrix} 1 & x & x^3 \\ 1-1 & b-x & b^3-x^3 \\ 1 & c & c^3 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & x & x^3 \\ 0 & b-x & b^3-x^3 \\ 1 & c & c^3 \end{vmatrix}$$

Applying $R_3 \to R_3 - R_1$, we get

$$\Delta = \begin{vmatrix} 1 & x & x^3 \\ 0 & b-x & b^3-x^3 \\ 1-1 & c-x & c^3-x^3 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & x & x^3 \\ 0 & b-x & b^3-x^3 \\ 0 & c-x & c^3-x^3 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & x & x^3 \\ 0 & b-x & (b-x)(b^2+bx+x^2) \\ 0 & c-x & (c-x)(c^2+cx+x^2) \end{vmatrix}$$

Taking (b – x) and (c – x) common from R_2 and R_3, we get

$$\Delta = (b-x)(c-x)\begin{vmatrix} 1 & x & x^3 \\ 0 & 1 & b^2+bx+x^2 \\ 0 & 1 & c^2+cx+x^2 \end{vmatrix}$$

Expanding the determinant along C_1, we have

$\Delta = (b - x)(c - x)(1)[(1)(c^2 + cx + x^2) - (1)(b^2 + bx + x^2)]$

$\Rightarrow \Delta = (b - x)(c - x)[c^2 + cx + x^2 - b^2 - bx - x^2]$

$\Rightarrow \Delta = (b - x)(c - x)[c^2 - b^2 + cx - bx]$

$\Rightarrow \Delta = (b - x)(c - x)[(c - b)(c + b) + (c - b)x]$

$\therefore \Delta = (b - x)(c - x)(c - b)(c + b + x)$

The given equation is $\Delta = 0$.

⇒ (b – x)(c – x)(c – b)(c + b + x) = 0

However, b ≠ c according to the given condition.

⇒ (b – x)(c – x)(c + b + x) = 0

Case – I:

b – x = 0 ⇒ x = b

Case – II:

c – x = 0 ⇒ x = c

Case – III:

c + b + x = 0 ⇒ x = –(b + c)

Thus, b, c and –(b + c) are the roots of the given determinant equation.

52 G. Question

Solve the following determinant equations:

$$\begin{vmatrix} 15-2x & 11-3x & 7-x \\ 11 & 17 & 14 \\ 10 & 16 & 13 \end{vmatrix} = 0$$

Answer

$$\begin{vmatrix} 15-2x & 11-3x & 7-x \\ 11 & 17 & 14 \\ 10 & 16 & 13 \end{vmatrix} = 0$$

Let $\Delta = \begin{vmatrix} 15-2x & 11-3x & 7-x \\ 11 & 17 & 14 \\ 10 & 16 & 13 \end{vmatrix}$

We need to find the roots of Δ = 0.

Recall that the value of a determinant remains same if we apply the operation $R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$.

Applying $R_2 \to R_2 - R_3$, we get

$$\Delta = \begin{vmatrix} 15-2x & 11-3x & 7-x \\ 11-10 & 17-16 & 14-13 \\ 10 & 16 & 13 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 15-2x & 11-3x & 7-x \\ 1 & 1 & 1 \\ 10 & 16 & 13 \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$, we get

$$\Delta = \begin{vmatrix} 15-2x & 11-3x-(15-2x) & 7-x \\ 1 & 1-1 & 1 \\ 10 & 16-10 & 13 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 15-2x & -4-x & 7-x \\ 1 & 0 & 1 \\ 10 & 6 & 13 \end{vmatrix}$$

Applying $C_3 \to C_3 - C_1$, we get

$$\Delta = \begin{vmatrix} 15-2x & -4-x & 7-x-(15-2x) \\ 1 & 0 & 1-1 \\ 10 & 6 & 13-10 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 15-2x & -4-x & x-8 \\ 1 & 0 & 0 \\ 10 & 6 & 3 \end{vmatrix}$$

Expanding the determinant along R_2, we have

Δ = – (1)[(–4 – x)(3) – (6)(x – 8)]

⇒ Δ = – [–12 – 3x – 6x + 48]

⇒ Δ = – [– 9x + 36]

∴ Δ = 9x – 36

The given equation is Δ = 0.

⇒ 9x – 36 = 0

⇒ 9x = 36

∴ x = 4

Thus, 4 is the root of the given determinant equation.

52 H. Question

Solve the following determinant equations:

$$\begin{vmatrix} 1 & 1 & x \\ p+1 & p+1 & p+x \\ 3 & x+1 & x+2 \end{vmatrix} = 0$$

Answer

$$\begin{vmatrix} 1 & 1 & x \\ p+1 & p+1 & p+x \\ 3 & x+1 & x+2 \end{vmatrix} = 0$$

Let $\Delta = \begin{vmatrix} 1 & 1 & x \\ p+1 & p+1 & p+x \\ 3 & x+1 & x+2 \end{vmatrix}$

We need to find the roots of Δ = 0.

Recall that the value of a determinant remains same if we apply the operation $R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$.

Applying $R_2 \to R_2 - R_1$, we get

$$\Delta = \begin{vmatrix} 1 & 1 & x \\ p+1-1 & p+1-1 & p+x-x \\ 3 & x+1 & x+2 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 1 & x \\ p & p & p \\ 3 & x+1 & x+2 \end{vmatrix}$$

Taking the term p common from R_2, we get

$$\Delta = p\begin{vmatrix} 1 & 1 & x \\ 1 & 1 & 1 \\ 3 & x+1 & x+2 \end{vmatrix}$$

Applying $C_1 \to C_1 - C_2$, we get

$$\Delta = p\begin{vmatrix} 1-1 & 1 & x \\ 1-1 & 1 & 1 \\ 3-(x+1) & x+1 & x+2 \end{vmatrix}$$

$$\Rightarrow \Delta = p\begin{vmatrix} 0 & 1 & x \\ 0 & 1 & 1 \\ 2-x & x+1 & x+2 \end{vmatrix}$$

Expanding the determinant along C_1, we have

$\Delta = p(2 - x)[(1)(1) - (1)(x)]$

$\therefore \Delta = p(2 - x)(1 - x)$

The given equation is $\Delta = 0$.

$\Rightarrow p(2 - x)(1 - x) = 0$

Assuming $p \neq 0$, we get

$\Rightarrow (2 - x)(1 - x) = 0$

Case – I:

$2 - x = 0 \Rightarrow x = 2$

Case – II:

$1 - x = 0 \Rightarrow x = 1$

Thus, 1 and 2 are the roots of the given determinant equation.

52 I. Question

Solve the following determinant equations:

$$\begin{vmatrix} 3 & -2 & \sin 3\theta \\ -7 & 8 & \cos 2\theta \\ -11 & 14 & 2 \end{vmatrix} = 0$$

Answer

$$\begin{vmatrix} 3 & -2 & \sin 3\theta \\ -7 & 8 & \cos 2\theta \\ -11 & 14 & 2 \end{vmatrix} = 0$$

Let $\Delta = \begin{vmatrix} 3 & -2 & \sin 3\theta \\ -7 & 8 & \cos 2\theta \\ -11 & 14 & 2 \end{vmatrix}$

We need to find the roots of $\Delta = 0$.

Recall that the value of a determinant remains same if we apply the operation $R_i \rightarrow R_i + kR_j$ or $C_i \rightarrow C_i + kC_j$.

Applying $C_1 \rightarrow C_1 + C_2$, we get

$$\Delta = \begin{vmatrix} 3+(-2) & -2 & \sin 3\theta \\ -7+8 & 8 & \cos 2\theta \\ -11+14 & 14 & 2 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & -2 & \sin 3\theta \\ 1 & 8 & \cos 2\theta \\ 3 & 14 & 2 \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_1$, we get

$$\Delta = \begin{vmatrix} 1 & -2 & \sin 3\theta \\ 1-1 & 8-(-2) & \cos 2\theta - \sin 3\theta \\ 3 & 14 & 2 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & -2 & \sin 3\theta \\ 0 & 10 & \cos 2\theta - \sin 3\theta \\ 3 & 14 & 2 \end{vmatrix}$$

Applying $R_3 \to R_3 - 3R_1$, we get

$$\Delta = \begin{vmatrix} 1 & -2 & \sin 3\theta \\ 0 & 10 & \cos 2\theta - \sin 3\theta \\ 3-3(1) & 14-3(-2) & 2-3(\sin 3\theta) \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & -2 & \sin 3\theta \\ 0 & 10 & \cos 2\theta - \sin 3\theta \\ 0 & 20 & 2-3\sin 3\theta \end{vmatrix}$$

Expanding the determinant along C_1, we have

$\Delta = (1)[(10)(2 - 3\sin(3\theta)) - (20)(\cos(2\theta) - \sin(3\theta))]$

$\Rightarrow \Delta = [20 - 30\sin(3\theta) - 20\cos(2\theta) + 20\sin(3\theta)]$

$\Rightarrow \Delta = 20 - 10\sin(3\theta) - 20\cos(2\theta)$

From trigonometry, we have $\sin(3\theta) = 3\sin\theta - 4\sin^3\theta$ and $\cos(2\theta) = 1 - 2\sin^2\theta$.

$\Rightarrow \Delta = 20 - 10(3\sin\theta - 4\sin^3\theta) - 20(1 - 2\sin^2\theta)$

$\Rightarrow \Delta = 20 - 30\sin\theta + 40\sin^3\theta - 20 + 40\sin^2\theta$

$\Rightarrow \Delta = -30\sin\theta + 40\sin^2\theta + 40\sin^3\theta$

$\therefore \Delta = 10(\sin\theta)(-3 + 4\sin\theta + 4\sin^2\theta)$

The given equation is $\Delta = 0$.

$\Rightarrow 10(\sin\theta)(-3 + 4\sin\theta + 4\sin^2\theta) = 0$

$\Rightarrow (\sin\theta)(-3 + 4\sin\theta + 4\sin^2\theta) = 0$

<u>Case – I</u>:

$\sin\theta = 0 \Rightarrow \theta = k\pi$, where $k \in Z$

<u>Case – II</u>:

$-3 + 4\sin\theta + 4\sin^2\theta = 0$

$\Rightarrow 4\sin^2\theta + 4\sin\theta - 3 = 0$

$\Rightarrow 4\sin^2\theta + 6\sin\theta - 2\sin\theta - 3 = 0$

$\Rightarrow 2\sin\theta(2\sin\theta + 3) - 1(2\sin\theta + 3) = 0$

$\Rightarrow (2\sin\theta - 1)(2\sin\theta + 3) = 0$

$\Rightarrow 2\sin\theta - 1 = 0$ or $2\sin\theta + 3 = 0$

$\Rightarrow 2\sin\theta = 1$ or $2\sin\theta = -3$

$\Rightarrow \sin\theta = \frac{1}{2}$ or $\sin\theta = -\frac{3}{2}$

However, $\sin\theta \neq -\frac{3}{2}$ as $-1 \leq \sin\theta \leq 1$.

$$\Rightarrow \sin\theta = \frac{1}{2} = \sin\frac{\pi}{6}$$

$\therefore \theta = k\pi + (-1)^k\frac{\pi}{6}$, where $k \in Z$

Thus, $k\pi$ and $k\pi + (-1)^k\frac{\pi}{6}$ for all integral values of k are the roots of the given determinant equation.

53. Question

If a, b and c are all non-zero and $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = 0$, then prove that $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1=0$.

Answer

Let $\Delta = \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$

Given that Δ = 0.

We can write the determinant Δ as

$$\Delta = \begin{vmatrix} a\left(\frac{1}{a}+1\right) & b\left(\frac{1}{b}\right) & c\left(\frac{1}{c}\right) \\ a\left(\frac{1}{a}\right) & b\left(\frac{1}{b}+1\right) & c\left(\frac{1}{c}\right) \\ a\left(\frac{1}{a}\right) & b\left(\frac{1}{b}\right) & c\left(\frac{1}{c}+1\right) \end{vmatrix}$$

Taking a, b and c common from C_1, C_2 and C_3, we get

$$\Rightarrow \Delta = (abc)\begin{vmatrix} 1+\frac{1}{a} & \frac{1}{b} & \frac{1}{c} \\ \frac{1}{a} & 1+\frac{1}{b} & \frac{1}{c} \\ \frac{1}{a} & \frac{1}{b} & 1+\frac{1}{c} \end{vmatrix}$$

Recall that the value of a determinant remains same if we apply the operation $R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$.

Applying $C_1 \to C_1 + C_2$, we get

$$\Delta = (abc)\begin{vmatrix} 1+\frac{1}{a}+\frac{1}{b} & \frac{1}{b} & \frac{1}{c} \\ \frac{1}{a}+\left(1+\frac{1}{b}\right) & 1+\frac{1}{b} & \frac{1}{c} \\ \frac{1}{a}+\frac{1}{b} & \frac{1}{b} & 1+\frac{1}{c} \end{vmatrix}$$

$$\Rightarrow \Delta = (abc)\begin{vmatrix} 1+\frac{1}{a}+\frac{1}{b} & \frac{1}{b} & \frac{1}{c} \\ 1+\frac{1}{a}+\frac{1}{b} & 1+\frac{1}{b} & \frac{1}{c} \\ \frac{1}{a}+\frac{1}{b} & \frac{1}{b} & 1+\frac{1}{c} \end{vmatrix}$$

Applying $C_1 \to C_1 + C_3$, we get

$$\Delta = (abc)\begin{vmatrix} 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & \frac{1}{b} & \frac{1}{c} \\ 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1+\frac{1}{b} & \frac{1}{c} \\ \frac{1}{a}+\frac{1}{b}+\left(1+\frac{1}{c}\right) & \frac{1}{b} & 1+\frac{1}{c} \end{vmatrix}$$

$$\Rightarrow \Delta = (abc)\begin{vmatrix} 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & \frac{1}{b} & \frac{1}{c} \\ 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1+\frac{1}{b} & \frac{1}{c} \\ 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & \frac{1}{b} & 1+\frac{1}{c} \end{vmatrix}$$

Taking $1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$ common from C_1, we get

$$\Delta=(abc)\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix}1 & \frac{1}{b} & \frac{1}{c}\\ 1 & 1+\frac{1}{b} & \frac{1}{c}\\ 1 & \frac{1}{b} & 1+\frac{1}{c}\end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$, we get

$$\Delta=(abc)\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix}1 & \frac{1}{b} & \frac{1}{c}\\ 1-1 & 1+\frac{1}{b}-\frac{1}{b} & \frac{1}{c}-\frac{1}{c}\\ 1 & \frac{1}{b} & 1+\frac{1}{c}\end{vmatrix}$$

$$\Rightarrow \Delta=(abc)\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix}1 & \frac{1}{b} & \frac{1}{c}\\ 0 & 1 & 0\\ 1 & \frac{1}{b} & 1+\frac{1}{c}\end{vmatrix}$$

Applying $R_3 \to R_3 - R_1$, we get

$$\Delta=(abc)\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix}1 & \frac{1}{b} & \frac{1}{c}\\ 0 & 1 & 0\\ 1-1 & \frac{1}{b}-\frac{1}{b} & 1+\frac{1}{c}-\frac{1}{c}\end{vmatrix}$$

$$\Rightarrow \Delta=(abc)\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix}1 & \frac{1}{b} & \frac{1}{c}\\ 0 & 1 & 0\\ 0 & 0 & 1\end{vmatrix}$$

Expanding the determinant along C_1, we have

$$\Delta=(abc)\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)(1)[(1)(1)-0]$$

$$\therefore \Delta=(abc)\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

We have $\Delta = 0$.

$$\Rightarrow (abc)\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)=0$$

It is given that a, b and c are all non-zero.

$$\therefore 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$$

Thus, $1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$ when $\begin{vmatrix}1+a & 1 & 1\\ 1 & 1+b & 1\\ 1 & 1 & 1+c\end{vmatrix}=0$ and a, b, c are all non-zero.

54. Question

If $\begin{vmatrix}a & b-y & c-z\\ a-x & b & c-z\\ a-x & b-y & c\end{vmatrix}=0$, then using properties of determinants, find the value of $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}$, where

$x, y, z \neq 0$.

Answer

Let $\Delta = \begin{vmatrix} a & b-y & c-z \\ a-x & b & c-z \\ a-x & b-y & c \end{vmatrix}$

Given that Δ = 0.

Recall that the value of a determinant remains same if we apply the operation $R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$.

Applying $R_2 \to R_2 - R_1$, we get

$$\Delta = \begin{vmatrix} a & b-y & c-z \\ a-x-a & b-(b-y) & c-z-(c-z) \\ a-x & b-y & c \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} a & b-y & c-z \\ -x & y & 0 \\ a-x & b-y & c \end{vmatrix}$$

Applying $R_3 \to R_3 - R_1$, we get

$$\Delta = \begin{vmatrix} a & b-y & c-z \\ -x & y & 0 \\ a-x-a & b-y-(b-y) & c-(c-z) \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} a & b-y & c-z \\ -x & y & 0 \\ -x & 0 & z \end{vmatrix}$$

Expanding the determinant along C_3, we have

$\Rightarrow \Delta = (c - z)[0 - (-x)(y)] - 0 + z[(a)(y) - (-x)(b - y)]$

$\Rightarrow \Delta = (c - z)(xy) + z[ay + xb - xy]$

$\Rightarrow \Delta = cxy - xyz + ayz + bxz - xyz$

$\therefore \Delta = ayz + bxz + cxy - 2xyz$

We have Δ = 0

$\Rightarrow ayz + bxz + cxy - 2xyz = 0$

$\Rightarrow ayz + bxz + cxy = 2xyz$

$$\Rightarrow \frac{ayz + bxz + cxy}{xyz} = 2$$

$$\Rightarrow \frac{ayz}{xyz} + \frac{bxz}{xyz} + \frac{cxy}{xyz} = 2$$

$$\therefore \frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$$

Thus, $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$ when $\begin{vmatrix} a & b-y & c-z \\ a-x & b & c-z \\ a-x & b-y & c \end{vmatrix} = 0$.

Exercise 6.3

1 A. Question

Find the area of the triangle with vertices at the points:

(3, 8), (– 4, 2) and (5, – 1)

Answer

Given: – Vertices of the triangle:

(3, 8), (– 4, 2) and (5, – 1)

We know that,

If vertices of a triangle are (x_1,y_1), (x_2,y_2) and (x_3,y_3), then the area of the triangle is given by:

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Now, substituting given value in above formula

$$\Delta = \frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 5 & -1 & 1 \end{vmatrix}$$

Expanding along R_1

$$= \frac{1}{2}\left[3\begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} - 8\begin{vmatrix} -4 & 1 \\ 5 & 1 \end{vmatrix} + 1\begin{vmatrix} -4 & 2 \\ 5 & -1 \end{vmatrix}\right]$$

$$= \frac{1}{2}[3(3) - 8(-9) + 1(-6)]$$

$$= \frac{1}{2}[9 + 72 - 6]$$

$= \frac{75}{2}$ sq.units

Thus area of triangle is $\frac{75}{2}$ sq.units

1 B. Question

Find the area of the triangle with vertices at the points:

(2, 7) (1, 1) and (10, 8)

Answer

Given: – Vertices of the triangle:

(2, 7) (1, 1) and (10, 8)

We know that,

If vertices of a triangle are (x_1,y_1), (x_2,y_2) and (x_3,y_3), then the area of the triangle is given by:

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Now, substituting given value in above formula

$$\Delta = \frac{1}{2}\begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$$

Expanding along R_1

$$= \frac{1}{2}\left[2\begin{vmatrix} 1 & 1 \\ 8 & 1 \end{vmatrix} - 7\begin{vmatrix} 1 & 1 \\ 10 & 1 \end{vmatrix} + 1\begin{vmatrix} 1 & 1 \\ 10 & 8 \end{vmatrix}\right]$$

$= \frac{1}{2}[2(-7) - 7(-9) + 1(-2)]$

$= \frac{1}{2}[-14 + 63 - 2]$

$= \frac{47}{2}$ sq.units

Thus area of triangle is $\frac{47}{2}$ sq.units

1 C. Question

Find the area of the triangle with vertices at the points:

(– 1, – 8), (– 2, – 3) and (3, 2)

Answer

Given: – Vertices of the triangle:

(– 1, – 8), (– 2, – 3) and (3, 2)

We know that,

If vertices of a triangle are (x_1,y_1), (x_2,y_2) and (x_3,y_3), then the area of the triangle is given by:

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Now, substituting given value in above formula

$$\Delta = \frac{1}{2}\begin{vmatrix} -1 & -8 & 1 \\ -2 & -3 & 1 \\ 3 & 2 & 1 \end{vmatrix}$$

Expanding along R_1

$= \frac{1}{2}\left[-1\begin{vmatrix} -3 & 1 \\ 2 & 1 \end{vmatrix} - 8\begin{vmatrix} -2 & 1 \\ 3 & 1 \end{vmatrix} + 1\begin{vmatrix} -2 & -3 \\ 3 & 2 \end{vmatrix}\right]$

$= \frac{1}{2}[-1(-5) - 8(-5) + 1(5)]$

$= \frac{1}{2}[5 - 40 + 5]$

$= \frac{-30}{2}$ sq.units

as area cannot be negative

Therefore, 15 sq.unit is the area

Thus area of triangle is 15 sq.units

1 D. Question

Find the area of the triangle with vertices at the points:

(0, 0) (6, 0) and (4, 3)

Answer

Given: – Vertices of the triangle:

(0, 0) (6, 0) and (4, 3)

We know that,

If vertices of a triangle are (x_1,y_1), (x_2,y_2) and (x_3,y_3), then the area of the triangle is given by:

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Now, substituting given value in above formula

$$\Delta = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}$$

Expanding along R_1

$$= \frac{1}{2}\left[0\begin{vmatrix} 0 & 1 \\ 3 & 1 \end{vmatrix} - 0\begin{vmatrix} 6 & 1 \\ 4 & 1 \end{vmatrix} + 1\begin{vmatrix} 6 & 0 \\ 4 & 3 \end{vmatrix}\right]$$

$$= \frac{1}{2}[0 - 0 + 1(18)]$$

$$= \frac{1}{2}[18]$$

= 9 sq.units

Thus area of triangle is 9 sq.units

2 A. Question

Using determinants show that the following points are collinear:

(5, 5), (– 5, 1) and (10, 7)

Answer

Given: – (5, 5), (– 5, 1) and (10, 7) are three points

Tip: – For Three points to be collinear, the area of the triangle formed by these points will be zero

Now, we know that,

vertices of a triangle are (x_1,y_1), (x_2,y_2) and (x_3,y_3), then the area of the triangle is given by:

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Now,

Substituting given value in above formula

$$\Delta = \frac{1}{2}\begin{vmatrix} 5 & 5 & 1 \\ -5 & 1 & 1 \\ 10 & 7 & 1 \end{vmatrix} = 0$$

R.H.S

$$\frac{1}{2}\begin{vmatrix} 5 & 5 & 1 \\ -5 & 1 & 1 \\ 10 & 7 & 1 \end{vmatrix}$$

Expanding along R_1

$$= \frac{1}{2}\left[5\begin{vmatrix} 1 & 1 \\ 7 & 1 \end{vmatrix} - 5\begin{vmatrix} -5 & 1 \\ 10 & 1 \end{vmatrix} + 1\begin{vmatrix} -5 & 1 \\ 10 & 7 \end{vmatrix}\right]$$

$$= \frac{1}{2}[5(-6) - 5(-15) + 1(-45)]$$

$= \frac{1}{2}[-35 + 75 - 45]$

= 0

= LHS

Since, Area of triangle is zero

Hence, points are collinear

2 B. Question

Using determinants show that the following points are collinear:

(1, – 1), (2, 1) and (4, 5)

Answer

Given: – (1, – 1), (2, 1) and (4, 5) are three points

Tip: – For Three points to be collinear, the area of the triangle formed by these points will be zero

Now, we know that,

vertices of a triangle are (x_1,y_1), (x_2,y_2) and (x_3,y_3), then the area of the triangle is given by:

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Now,

Substituting given value in above formula

$$\Delta = \frac{1}{2}\begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & 1 \\ 4 & 5 & 1 \end{vmatrix} = 0$$

R.H.S

$$\frac{1}{2}\begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & 1 \\ 4 & 5 & 1 \end{vmatrix}$$

Expanding along R_1

$$= \frac{1}{2}\left[1\begin{vmatrix} 1 & 1 \\ 5 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ 4 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ 4 & 5 \end{vmatrix}\right]$$

$$= \frac{1}{2}[1 - 5 + 2 - 4 + 10 - 4]$$

$$= \frac{1}{2}[0]$$

= 0

= LHS

Since, Area of triangle is zero.

Hence, points are collinear.

2 C. Question

Using determinants show that the following points are collinear:

(3, – 2), (8, 8) and (5, 2)

Answer

Given: – (3, – 2), (8, 8) and (5, 2) are three points

Tip: – For Three points to be collinear, the area of triangle formed by these points will be zero

Now, we know that,

vertices of a triangle are (x_1,y_1), (x_2,y_2) and (x_3,y_3), then the area of the triangle is given by:

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Now,

Substituting given value in above formula

$$\Delta = \frac{1}{2}\begin{vmatrix} 3 & -2 & 1 \\ 8 & 8 & 1 \\ 5 & 2 & 1 \end{vmatrix} = 0$$

R.H.S

$$\frac{1}{2}\begin{vmatrix} 3 & -2 & 1 \\ 8 & 8 & 1 \\ 5 & 2 & 1 \end{vmatrix}$$

Expanding along R_1

$$= \frac{1}{2}\left[3\begin{vmatrix} 8 & 1 \\ 2 & 1 \end{vmatrix} - 2\begin{vmatrix} 8 & 1 \\ 5 & 1 \end{vmatrix} + 1\begin{vmatrix} 8 & 8 \\ 5 & 2 \end{vmatrix}\right]$$

$$= \frac{1}{2}[3(6) - 2(3) + 1(-24)]$$

$$= \frac{1}{2}[0]$$

= 0

= LHS

Since, Area of triangle is zero

Hence, points are collinear.

2 D. Question

Using determinants show that the following points are collinear:

(2, 3), (– 1, – 2) and (5, 8)

Answer

Given: – (2, 3), (– 1, – 2) and (5, 8) are three points

Tip: – For Three points to be collinear, the area of the triangle formed by these points will be zero

Now, we know that,

vertices of a triangle are (x_1,y_1), (x_2,y_2) and (x_3,y_3), then the area of the triangle is given by:

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Now,

Substituting given value in above formula

$$\Delta = \frac{1}{2}\begin{vmatrix} 2 & 3 & 1 \\ -1 & -2 & 1 \\ 5 & 8 & 1 \end{vmatrix} = 0$$

R.H.S

$$\frac{1}{2}\begin{vmatrix} 2 & 3 & 1 \\ -1 & -2 & 1 \\ 5 & 8 & 1 \end{vmatrix}$$

Expanding along R_1

$$= \frac{1}{2}\left[2\begin{vmatrix} -2 & 1 \\ 8 & 1 \end{vmatrix} - 3\begin{vmatrix} -1 & 1 \\ 5 & 1 \end{vmatrix} + 1\begin{vmatrix} -1 & -2 \\ 5 & 8 \end{vmatrix}\right]$$

$$= \frac{1}{2}[2(-10) - 3(-1-5) + 1(-8+10)]$$

$$= \frac{1}{2}[-20 + 18 + 2]$$

= 0

= LHS

Since, Area of triangle is zero

Hence, points are collinear.

3. Question

If the points (a, 0), (o, b) and (1, 1) are collinear, prove that a + b = ab.

Answer

Given: – (a, 0), (o, b) and (1, 1) are collinear points

To Prove: – a + b = ab

Proof: –

Tip: – If Three points to be collinear, then the area of the triangle formed by these points will be zero

Now, we know that,

vertices of a triangle are (x_1,y_1), (x_2,y_2) and (x_3,y_3), then the area of the triangle is given by:

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Thus

$$\frac{1}{2}\begin{vmatrix} a & 0 & 1 \\ 0 & b & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

Expanding along R_1

$$\Rightarrow 0 = \frac{1}{2}\left[a\begin{vmatrix} b & 1 \\ 1 & 1 \end{vmatrix} - 0\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 0 & b \\ 1 & 1 \end{vmatrix}\right]$$

$$\Rightarrow \frac{1}{2}[a(b-1) - 0(-1) + 1(-b)] = 0$$

$$\Rightarrow \frac{1}{2}[ab - a - b] = 0$$

⇒ a + b = ab

Hence Proved

4. Question

Using determinants prove that the points (a, b) (a', b') and (a – a', b – b') are collinear if ab' = a'b.

Answer

Given: – (a, b) (a', b') and (a – a', b – b') are points and ab' = a'b

To Prove: – (a, b) (a', b') and (a – a', b – b') are collinear points

Proof: –

Tip: – If three points to be collinear, then the area of the triangle formed by these points will be zero.

Now, we know that,

vertices of a triangle are (x_1,y_1), (x_2,y_2) and (x_3,y_3), then the area of the triangle is given by:

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Thus

$$\frac{1}{2}\begin{vmatrix} a & b & 1 \\ a' & b' & 1 \\ a-a' & b-b' & 1 \end{vmatrix} = 0$$

Expanding along R_1

$$\Rightarrow 0 = \frac{1}{2}\left[a\begin{vmatrix} b' & 1 \\ b-b' & 1 \end{vmatrix} - b\begin{vmatrix} a' & 1 \\ a-a' & 1 \end{vmatrix} + 1\begin{vmatrix} a' & b' \\ a-a' & b-b' \end{vmatrix}\right]$$

$$\Rightarrow \frac{1}{2}\left[a(b'-b+b') - b(a'-a+a') + 1(a'b - a'b' - ab' + a'b')\right] = 0$$

$$\Rightarrow \frac{1}{2}\left[a'b - ab + ab' - a'b + ab + a'b + a'b - a'b' - ab' + a'b'\right] = 0$$

$\Rightarrow ab' - a'b = 0$

$\Rightarrow ab' = a'b$

Hence, Proved.

5. Question

Find the value of λ so that the points (1, – 5), (– 4, 5) and (λ, 7) are collinear.

Answer

Given: – (1, – 5), (– 4, 5) and (λ, 7) are collinear

Tip: – For Three points to be collinear, the area of the triangle formed by these points will be zero

Now, we know that,

vertices of a triangle are (x_1,y_1), (x_2,y_2) and (x_3,y_3), then the area of the triangle is given by:

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Now,

Substituting given value in above formula

$$\Delta = \frac{1}{2}\begin{vmatrix} 1 & -5 & 1 \\ -4 & 5 & 1 \\ \lambda & 7 & 1 \end{vmatrix} = 0$$

Expanding along R_1

$\Rightarrow \frac{1}{2}\left[1\begin{vmatrix}5 & 1\\ 7 & 1\end{vmatrix} + 5\begin{vmatrix}-4 & 1\\ \lambda & 1\end{vmatrix} + 1\begin{vmatrix}-4 & 5\\ \lambda & 7\end{vmatrix}\right] = 0$

$\Rightarrow \frac{1}{2}[1(-2) + 5(-4-\lambda) + 1(-28-5\lambda)] = 0$

$\Rightarrow \frac{1}{2}[-2-20-5\lambda-28-5\lambda] = 0$

$\Rightarrow -50 - 10\lambda = 0$

$\Rightarrow \lambda = -5$

$\Rightarrow$

6. Question

Find the value of x if the area of a triangle is 35 square cms with vertices (x, 4), (2, – 6) and (5, 4).

Answer

Given: – Vertices of triangle are (x, 4), (2, – 6) and (5, 4) and area of triangle is 35 sq.cms

Tip: – If vertices of a triangle are (x_1,y_1), (x_2,y_2) and (x_3,y_3), then the area of the triangle is given by:

$$\Delta = \frac{1}{2}\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}$$

Now,

Substituting given value in above formula

$\Rightarrow 35 = \left|\frac{1}{2}\begin{vmatrix}x & 4 & 1\\ 2 & -6 & 1\\ 5 & 4 & 1\end{vmatrix}\right|$

Removing modulus

$\Rightarrow \pm 2 \times 35 = \begin{vmatrix}x & 4 & 1\\ 2 & -6 & 1\\ 5 & 4 & 1\end{vmatrix}$

Expanding along R_1

$\Rightarrow \left[x\begin{vmatrix}-6 & 1\\ 4 & 1\end{vmatrix} - 4\begin{vmatrix}2 & 1\\ 5 & 1\end{vmatrix} + 1\begin{vmatrix}2 & -6\\ 5 & 4\end{vmatrix}\right] = \pm 70$

$\Rightarrow [x(-10) - 4(-3) + 1(8 - 30)] = \pm 70$

$\Rightarrow [-10x + 12 + 38] = \pm 70$

$\Rightarrow \pm 70 = -10x + 50$

Taking + ve sign, we get

$\Rightarrow +70 = -10x + 50$

$\Rightarrow 10x = -20$

$\Rightarrow x = -2$

Taking – ve sign, we get

$\Rightarrow -70 = -10x + 50$

$\Rightarrow 10x = 120$

$\Rightarrow x = 12$

Thus x = – 2, 12

7. Question

Using determinants, find the area of a triangle whose vertices are (1, 4), (2, 3) and (– 5, – 3). Are the given points collinear?

Answer

Given: – Vertices are (1, 4), (2, 3) and (– 5, – 3)

Tip: – If vertices of a triangle are (x_1,y_1), (x_2,y_2) and (x_3,y_3), then the area of the triangle is given by:

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Now,

Substituting given value in above formula

$$\Delta = \frac{1}{2}\begin{vmatrix} 1 & 4 & 1 \\ 2 & 3 & 1 \\ -5 & -3 & 1 \end{vmatrix} = 0$$

Expanding along R_1

$$\Rightarrow \Delta = \frac{1}{2}\left[1\begin{vmatrix} 3 & 1 \\ -3 & 1 \end{vmatrix} - 4\begin{vmatrix} 3 & 1 \\ -3 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 3 \\ -5 & -3 \end{vmatrix}\right]$$

$$\Rightarrow \frac{1}{2}\left[1(6) - 4(7) + 1(9)\right] = \Delta$$

$$\Rightarrow \frac{1}{2}[-13] = \Delta$$

Since area can’t be negative

$$\Rightarrow \Delta = \frac{13}{2}$$

Tip: – For Three points to be collinear, the area of the triangle formed by these points will be zero

Now, as the area is not zero

Therefore, Points (1, 4), (2, 3) and (– 5, – 3) are not collinear.

8. Question

Using determinants, find the area of the triangle with vertices (– 3, 5), (3, – 6) and (7, 2).

Answer

Given: – Vertices are (– 3, 5), (3, – 6) and (7, 2)

Tip: – If vertices of a triangle are (x_1,y_1), (x_2,y_2) and (x_3,y_3), then the area of the triangle is given by:

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Now,

Substituting given value in above formula

$$\Delta = \frac{1}{2}\begin{vmatrix} -3 & 5 & 1 \\ 3 & -6 & 1 \\ 7 & 2 & 1 \end{vmatrix}$$

Expanding along R_1

$\Rightarrow \Delta = \frac{1}{2}\left[-3\begin{vmatrix}-6 & 1\\ 2 & 1\end{vmatrix} - 5\begin{vmatrix}3 & 1\\ 7 & 1\end{vmatrix} + 1\begin{vmatrix}3 & -6\\ 7 & 2\end{vmatrix}\right]$

$\Rightarrow \frac{1}{2}[-3(-8) - 5(-4) + 1(48)] = \Delta$

$\Rightarrow \frac{1}{2}[24 + 20 + 48] = \Delta$

$\Rightarrow \Delta = \frac{92}{2}$

$\Rightarrow \Delta = 46$ sq. units

9. Question

Using determinants, find the value of k so that the points (k, 2 – 2 k), (– k + 1, 2k) and (– 4 – k, 6 – 2k) may be collinear.

Answer

Given: – Points are (k, 2 – 2 k), (– k + 1, 2k) and (– 4 – k, 6 – 2k) which are collinear

Tip: – For Three points to be collinear, the area of the triangle formed by these points will be zero.

If vertices of a triangle are (x_1,y_1), (x_2,y_2) and (x_3,y_3), then the area of the triangle is given by:

$$\Delta = \frac{1}{2}\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}$$

Now,

Substituting given value in above formula

$$\Delta = \frac{1}{2}\begin{vmatrix}k & 2-2k & 1\\ -k+1 & 2k & 1\\ -4-k & 6-2k & 1\end{vmatrix} = 0$$

Expanding along R_1

$\Rightarrow \frac{1}{2}\left[k\begin{vmatrix}2k & 1\\ 6-2k & 1\end{vmatrix} - (2-2k)\begin{vmatrix}-k+1 & 1\\ -4-k & 1\end{vmatrix} + 1\begin{vmatrix}-k+1 & 2k\\ -4-k & 6-2k\end{vmatrix}\right] = 0$

$\Rightarrow k(2k - 6 + 2k) - (2 - 2k)(-k + 1 + 4 + k) + 1(6 - 2k - 6k + 2k^2 + 8k + 2k^2) = 0$

$\Rightarrow 4k^2 - 6k - 10 + 10k + 6 + 4k^2 = 0$

$\Rightarrow 8k^2 + 4k - 4 = 0$

$\Rightarrow 8k^2 + 8k - 4k - 4 = 0$

$\Rightarrow 8k(k + 1) - 4(k + 1) = 0$

$\Rightarrow (8k - 4)(k + 1) = 0$

If 8k – 4 = 0

$\Rightarrow k = \frac{1}{2}$

And, If k + 1 = 0

$\Rightarrow K = -1$

Hence, k = – 1, 0.5

10. Question

If the points (x, 2), (5, – 2) and (8, 8) are collinear, find x using determinants.

Answer

Given: – (x, 2), (5, – 2) and (8, 8) are collinear points

Tip: – For Three points to be collinear, the area of the triangle formed by these points will be zero.

Now, we know that,

Vertices of a triangle are (x_1,y_1), (x_2,y_2) and (x_3,y_3), then the area of the triangle is given by:

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Now,

Substituting given value in above formula

$$\Delta = \frac{1}{2}\begin{vmatrix} x & -2 & 1 \\ 5 & 2 & 1 \\ 8 & 8 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x & -2 & 1 \\ 5 & 2 & 1 \\ 8 & 8 & 1 \end{vmatrix} = 0$$

Expanding along R_1

$$\Rightarrow \left[x\begin{vmatrix} 2 & 1 \\ 8 & 1 \end{vmatrix} + 2\begin{vmatrix} 5 & 1 \\ 8 & 1 \end{vmatrix} + 1\begin{vmatrix} 5 & 2 \\ 8 & 8 \end{vmatrix}\right] = 0$$

$\Rightarrow [x(-6) + 2(-3) + 1(24)] = 0$

$\Rightarrow -6x - 6 + 24 = 0$

$\Rightarrow x = 3$

11. Question

If the points (3, – 2), (x,2) and (8,8) are collinear, find x using determinant.

Answer

Given: – (3, – 2), (x,2) and (8,8) are collinear points

Tip: – For Three points to be collinear, the area of the triangle formed by these points will be zero

Now, we know that,

Vertices of a triangle are (x_1,y_1), (x_2,y_2) and (x_3,y_3), then the area of the triangle is given by:

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Now,

Substituting given value in above formula

$$\Delta = \frac{1}{2}\begin{vmatrix} 3 & -2 & 1 \\ x & 2 & 1 \\ 8 & 8 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 3 & -2 & 1 \\ x & 2 & 1 \\ 8 & 8 & 1 \end{vmatrix} = 0$$

Expanding along R_1

$$\Rightarrow \left[x\begin{vmatrix} 2 & 1 \\ 8 & 1 \end{vmatrix} + 2\begin{vmatrix} x & 1 \\ 8 & 1 \end{vmatrix} + 1\begin{vmatrix} x & 2 \\ 8 & 8 \end{vmatrix}\right] = 0$$

$\Rightarrow [x(-6) + 2(x - 8) + 1(8x - 16)] = 0$

⇒ – 6x + 2x – 16 + 8x – 16 = 0

⇒ 10x = 50

⇒ x = 5

12 A. Question

Using determinants, find the equation of the line joining the points

(1, 2) and (3, 6)

Answer

Given: – (1, 2) and (3, 6) are collinear points

Tip: – For Three points to be collinear, the area of the triangle formed by these points will be zero

Now, we know that,

Vertices of a triangle are (x_1,y_1), (x_2,y_2) and (x_3,y_3), then the area of the triangle is given by:

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Now,

Let, 3^{rd} point be (x,y)

Substituting given value in above formula

$$\Delta = \frac{1}{2}\begin{vmatrix} x & y & 1 \\ 1 & 2 & 1 \\ 3 & 6 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x & y & 1 \\ 1 & 2 & 1 \\ 3 & 6 & 1 \end{vmatrix} = 0$$

Expanding along R_1

$$\Rightarrow \left[x\begin{vmatrix} 2 & 1 \\ 6 & 1 \end{vmatrix} - y\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} + 1\begin{vmatrix} 1 & 2 \\ 3 & 6 \end{vmatrix}\right] = 0$$

⇒ [x(– 4) – y(– 2) + 1(0)] = 0

⇒ – 4x + 2y = 0

⇒ y = 2x

It’s the equation of line

12 B. Question

Using determinants, find the equation of the line joining the points

(3, 1) and (9, 3)

Answer

Given: – (3, 1) and (9, 3) are collinear points

Tip: – For Three points to be collinear, the area of triangle formed by these points will be zero

Now, we know that,

Vertices of a triangle are (x_1,y_1), (x_2,y_2) and (x_3,y_3), then the area of the triangle is given by:

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Now,

Let, 3^{rd} point be (x,y)

Substituting given value in above formula

$$\Delta = \frac{1}{2}\begin{vmatrix} x & y & 1 \\ 3 & 1 & 1 \\ 9 & 3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x & y & 1 \\ 3 & 1 & 1 \\ 9 & 3 & 1 \end{vmatrix} = 0$$

Expanding along R_1

$$\Rightarrow \left[x\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} - y\begin{vmatrix} 3 & 1 \\ 9 & 1 \end{vmatrix} + 1\begin{vmatrix} 1 & 3 \\ 3 & 9 \end{vmatrix}\right] = 0$$

⇒ [x(– 2) – y(– 6) + 1(0)] = 0

⇒ – 2x + 6y = 0

⇒ x – 3y = 0

It’s the equation of line

13 A. Question

Find values of K, if the area of a triangle is 4 square units whose vertices are

(k,0), (4,0) and (0,2)

Answer

Given: – Vertices of triangle are (k, 0), (4, 0) and (0, 2) and area of triangle is 4 sq. units

Tip: – If vertices of a triangle are (x_1,y_1), (x_2,y_2) and (x_3,y_3), then the area of the triangle is given by:

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Now,

Substituting given value in above formula

$$\Rightarrow 4 = \left|\frac{1}{2}\begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix}\right|$$

Removing modulus

$$\Rightarrow \pm 2 \times 4 = \begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix}$$

Expanding along R_1

$$\Rightarrow \left[k\begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix} - 0\begin{vmatrix} 4 & 1 \\ 0 & 1 \end{vmatrix} + 1\begin{vmatrix} 4 & 0 \\ 0 & 2 \end{vmatrix}\right] = \pm 8$$

⇒ [k(– 2) – 0(4) + 1(8 – 0)] = ±8

⇒ [– 2k + 8] = ± 8

Taking + ve sign, we get

⇒ + 8 = – 2x + 8

⇒ – 2k = 0

⇒ k = 0

Taking – ve sign, we get

⇒ – 8 = – 2x + 8

⇒ – 2x = – 16

⇒ x = 8

Thus x = 0, 8

13 B. Question

Find values of K, if the area of a triangle is 4 square units whose vertices are

(– 2,0), (0, 4) and (0, k)

Answer

Given: – Vertices of triangle are (– 2,0), (0, 4) and (0, k) and the area of the triangle is 4 sq. units.

Tip: – If vertices of a triangle are (x_1,y_1), (x_2,y_2) and (x_3,y_3), then the area of the triangle is given by:

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Now,

Substituting given value in above formula

$$\Rightarrow 4 = \left|\frac{1}{2}\begin{vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{vmatrix}\right|$$

Removing modulus

$$\Rightarrow \pm 2 \times 4 = \begin{vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{vmatrix}$$

Expanding along R_1

$$\Rightarrow \left[-2\begin{vmatrix} 4 & 1 \\ k & 1 \end{vmatrix} - 0\begin{vmatrix} 0 & 1 \\ 0 & 1 \end{vmatrix} + 1\begin{vmatrix} 0 & 4 \\ 0 & k \end{vmatrix}\right] = \pm 8$$

⇒ [– 2(4 – k) – 0(0) + 1(0 – 0)] = ±8

⇒ – 8 + 2k = ± 8

Taking + ve sign, we get

⇒ 8 = – 8 + 2k

⇒ 2k = 16

⇒ k = 8

Taking – ve sign, we get

⇒ – 8 = 2x – 8

⇒ 2k = 0

⇒ k = 0

Thus k = 0, 8

Exercise 6.4

1. Question

Solve the following systems of linear equations by Cramer's rule:

x – 2y = 4

– 3x + 5y = – 7

Answer

Given: - Two equations x – 2y = 4 and – 3x + 5y = – 7

Tip: - Theorem - Cramer's Rule

Let there be a system of n simultaneous linear equations and with n unknown given by

$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$

$\vdots \vdots \vdots$

$a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n1} & \cdots & a_{nn} \end{vmatrix}$$

and let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \ldots, x_n = \frac{D_n}{D}$ provided that D ≠ 0

Now, here we have

x – 2y = 4

– 3x + 5y = – 7

So by comparing with the theorem, let's find D, D_1 and D_2

$$\Rightarrow D = \begin{vmatrix} 1 & -2 \\ -3 & 5 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} row

⇒ D = 5(1) – (– 3)(– 2)

⇒ D = 5 – 6

⇒ D = – 1

Again,

$$\Rightarrow D_1 = \begin{vmatrix} 4 & -2 \\ -7 & 5 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} row

$\Rightarrow D_1 = 5(4) - (-7)(-2)$

$\Rightarrow D_1 = 20 - 14$

$\Rightarrow D_1 = 6$

and

$$\Rightarrow D_2 = \begin{vmatrix} 1 & 4 \\ -3 & -7 \end{vmatrix}$$

Solving determinant, expanding along 1st row

$\Rightarrow D_2 = 1(-7) - (-3)(4)$

$\Rightarrow D_2 = -7 + 12$

$\Rightarrow D_2 = 5$

Thus by Cramer's Rule, we have

$$\Rightarrow x = \frac{D_1}{D}$$

$$\Rightarrow x = \frac{6}{-1}$$

$\Rightarrow x = -6$

and

$$\Rightarrow y = \frac{D_2}{D}$$

$$\Rightarrow y = \frac{5}{-1}$$

$\Rightarrow y = -5$

2. Question

Solve the following systems of linear equations by Cramer's rule:

$2x - y = 1$

$7x - 2y = -7$

Answer

Given: - Two equations $2x - y = 1$ and $7x - 2y = -7$

Tip: - Theorem – Cramer's Rule

Let there be a system of n simultaneous linear equations and with n unknown given by

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$$

$$\vdots \;\vdots\; \vdots$$

$$a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n$$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n1} & \ldots & a_{nn} \end{vmatrix}$$

and let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \ldots, x_n = \frac{D_n}{D}$ provided that $D \neq 0$

Now, here we have

$2x - y = 1$

$7x - 2y = -7$

So by comparing with the theorem, let's find D, D_1 and D_2

$\Rightarrow D = \begin{vmatrix} 2 & -1 \\ 7 & -2 \end{vmatrix}$

Solving determinant, expanding along 1st row

$\Rightarrow D = 2(-2) - (7)(-1)$

$\Rightarrow D = -4 + 7$

$\Rightarrow D = 3$

Again,

$\Rightarrow D_1 = \begin{vmatrix} 1 & -1 \\ -7 & -2 \end{vmatrix}$

Solving determinant, expanding along 1st row

$\Rightarrow D_1 = 1(-2) - (-7)(-1)$

$\Rightarrow D_1 = -2 - 7$

$\Rightarrow D_1 = -9$

and

$\Rightarrow D_2 = \begin{vmatrix} 2 & 1 \\ 7 & -7 \end{vmatrix}$

Solving determinant, expanding along 1st row

$\Rightarrow D_2 = 2(-7) - (7)(1)$

$\Rightarrow D_2 = -14 - 7$

$\Rightarrow D_2 = -21$

Thus by Cramer's Rule, we have

$\Rightarrow x = \frac{D_1}{D}$

$\Rightarrow x = \frac{-9}{3}$

$\Rightarrow x = -3$

and

$\Rightarrow y = \frac{D_2}{D}$

$\Rightarrow y = \frac{-21}{3}$

$\Rightarrow y = -7$

3. Question

Solve the following systems of linear equations by Cramer's rule:

$2x - y = 17$

$3x + 5y = 6$

Answer

Given: - Two equations $2x - y = 17$ and $3x + 5y = 6$

Tip: - Theorem – Cramer's Rule

Let there be a system of n simultaneous linear equations and with n unknown given by

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

$$\vdots \; \vdots \; \vdots$$

$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n1} & \dots & a_{nn} \end{vmatrix}$$

and let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \dots, x_n = \frac{D_n}{D}$ provided that $D \neq 0$

Now, here we have

$2x - y = 17$

$3x + 5y = 6$

So by comparing with the theorem, let's find D, D_1 and D_2

$$\Rightarrow D = \begin{vmatrix} 2 & -1 \\ 3 & 5 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} row

$\Rightarrow D = 2(5) - (3)(-1)$

$\Rightarrow D = 10 + 3$

$\Rightarrow D = 13$

Again,

$$\Rightarrow D_1 = \begin{vmatrix} 17 & -1 \\ 6 & 5 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} row

$\Rightarrow D_1 = 17(5) - (6)(-1)$

$\Rightarrow D_1 = 85 + 6$

$\Rightarrow D_1 = 91$

and

$$\Rightarrow D_2 = \begin{vmatrix} 2 & 17 \\ 3 & 6 \end{vmatrix}$$

Solving determinant, expanding along 1st row

$\Rightarrow D_2 = 2(6) - (17)(3)$

$\Rightarrow D_2 = 12 - 51$

$\Rightarrow D_2 = -39$

Thus by Cramer's Rule, we have

$$\Rightarrow x = \frac{D_1}{D}$$

$$\Rightarrow x = \frac{91}{13}$$

$\Rightarrow x = 7$

and

$$\Rightarrow y = \frac{D_2}{D}$$

$$\Rightarrow y = \frac{-39}{13}$$

$\Rightarrow y = -3$

4. Question

Solve the following systems of linear equations by Cramer's rule:

$3x + y = 19$

$3x - y = 23$

Answer

Given: - Two equations $3x + y = 19$ and $3x - y = 23$

Tip: - Theorem – Cramer's Rule

Let there be a system of n simultaneous linear equations and with n unknown given by

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

$$\vdots \; \vdots \; \vdots$$

$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n1} & \dots & a_{nn} \end{vmatrix}$$

and let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \ldots, x_n = \frac{D_n}{D}$ provided that $D \neq 0$

Now, here we have

3x + y = 19

3x – y = 23

So by comparing with the theorem, let's find D, D_1 and D_2

$$\Rightarrow \ D = \begin{vmatrix} 3 & 1 \\ 3 & -1 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} row

⇒ D = 3(– 1) – (3)(1)

⇒ D = – 3 – 3

⇒ D = – 6

Again,

$$\Rightarrow \ D_1 = \begin{vmatrix} 19 & 1 \\ 23 & -1 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} row

⇒ D_1 = 19(– 1) – (23)(1)

⇒ D_1 = – 19 – 23

⇒ D_1 = – 42

and

$$\Rightarrow \ D_2 = \begin{vmatrix} 3 & 19 \\ 3 & 23 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} row

⇒ D_2 = 3(23) – (19)(3)

⇒ D_2 = 69 – 57

⇒ D_2 = 12

Thus by Cramer's Rule, we have

$$\Rightarrow x = \frac{D_1}{D}$$

$$\Rightarrow x = \frac{-42}{-6}$$

⇒ x = 7

and

$$\Rightarrow y = \frac{D_2}{D}$$

$$\Rightarrow y = \frac{12}{-6}$$

$\Rightarrow$ y = – 2

5. Question

Solve the following systems of linear equations by Cramer's rule:

2x – y = – 2

3x + 4y = 3

Answer

Given : – Two equations 2x – y = – 2 and 3x + 4y = 3

Tip: - Theorem – Cramer's Rule

Let there be a system of n simultaneous linear equations and with n unknown given by

$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$

$\vdots \ \vdots \ \vdots$

$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n1} & \dots & a_{nn} \end{vmatrix}$$

and let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \dots, x_n = \frac{D_n}{D}$ provided that $D \neq 0$

Now, here we have

2x – y = – 2

3x + 4y = 3

So by comparing with the theorem, let's find D, D_1 and D_2

$$\Rightarrow \quad D = \begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} row

$\Rightarrow$ D = 2(4) – (3)(– 1)

$\Rightarrow$ D = 8 + 3

$\Rightarrow$ D = 11

Again,

$$\Rightarrow \quad D_1 = \begin{vmatrix} -2 & -1 \\ 3 & 4 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} row

$\Rightarrow$ D_1 = – 2(4) – (3)(– 1)

$\Rightarrow D_1 = -8 + 3$

$\Rightarrow D_1 = -5$

and

$$\Rightarrow \quad D_2 = \begin{vmatrix} 2 & -2 \\ 3 & 3 \end{vmatrix}$$

Solving determinant, expanding along 1st row

$\Rightarrow D_2 = 3(2) - (-2)(3)$

$\Rightarrow D_2 = 6 + 6$

$\Rightarrow D_2 = 12$

Thus by Cramer's Rule, we have

$\Rightarrow x = \frac{D_1}{D}$

$\Rightarrow x = \frac{-5}{11}$

and

$\Rightarrow y = \frac{D_2}{D}$

$\Rightarrow y = \frac{12}{11}$

6. Question

Solve the following systems of linear equations by Cramer's rule:

$3x + ay = 4$

$2x + ay = 2, a \neq 0$

Answer

Given: - Two equations $3x + ay = 4$ and $2x + ay = 2$, $a \neq 0$

Tip: - Theorem – Cramer's Rule

Let there be a system of n simultaneous linear equations and with n unknown given by

$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$

$\vdots \; \vdots \; \vdots$

$a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n1} & \ldots & a_{nn} \end{vmatrix}$$

and let D_j be the determinant obtained from D after replacing the jth column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \ldots, x_n = \frac{D_n}{D}$ provided that D ≠ 0

Now, here we have

3x + ay = 4

2x + ay = 2, a≠0

So by comparing with the theorem, let's find D, D_1 and D_2

$\Rightarrow D = \begin{vmatrix} 3 & a \\ 2 & a \end{vmatrix}$

Solving determinant, expanding along 1st row

⇒ D = 3(a) – (2)(a)

⇒ D = 3a – 2a

⇒ D = a

Again,

$\Rightarrow D_1 = \begin{vmatrix} 4 & a \\ 2 & a \end{vmatrix}$

Solving determinant, expanding along 1st row

⇒ D_1 = 4(a) – (2)(a)

⇒ D = 4a – 2a

⇒ D = 2a

and

$\Rightarrow D_2 = \begin{vmatrix} 3 & 4 \\ 2 & 2 \end{vmatrix}$

Solving determinant, expanding along 1st row

⇒ D_2 = 3(2) – (2)(4)

⇒ D = 6 – 8

⇒ D = – 2

Thus by Cramer’s Rule, we have

$\Rightarrow x = \frac{D_1}{D}$

$\Rightarrow x = \frac{2a}{a}$

⇒ x = 2

and

$\Rightarrow y = \frac{D_2}{D}$

$\Rightarrow y = \frac{-2}{a}$

7. Question

Solve the following systems of linear equations by Cramer’s rule:

2x + 3y = 10

x + 6y = 4

Answer

Given: - Two equations 2x – 3y = 10 and x + 6y = 4

Tip: - Theorem – Cramer's Rule

Let there be a system of n simultaneous linear equations and with n unknown given by

$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$

$\vdots \vdots \vdots$

$a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n1} & \ldots & a_{nn} \end{vmatrix}$$

and let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \ldots, x_n = \frac{D_n}{D}$ provided that $D \neq 0$

Now, here we have

2x + 3y = 10

x + 6y = 4

So by comparing with the theorem, let's find D, D_1 and D_2

$$\Rightarrow D = \begin{vmatrix} 2 & 3 \\ 1 & 6 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} row

⇒ D = 2(6) – (3)(1)

⇒ D = 12 – 3

⇒ D = 9

Again,

$$\Rightarrow D_1 = \begin{vmatrix} 10 & 3 \\ 4 & 6 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} row

⇒ D_1 = 10(6) – (3)(4)

⇒ D = 60 – 12

⇒ D = 48

and

$$\Rightarrow D_2 = \begin{vmatrix} 2 & 10 \\ 1 & 4 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} row

⇒ $D_2 = 2(4) - (10)(1)$

⇒ $D_2 = 8 - 10$

⇒ $D_2 = -2$

Thus by Cramer's Rule, we have

⇒ $x = \frac{D_1}{D}$

⇒ $x = \frac{48}{9}$

⇒ $x = \frac{16}{3}$

and

⇒ $y = \frac{D_2}{D}$

⇒ $y = \frac{-2}{9}$

⇒ $y = \frac{-2}{9}$

8. Question

Solve the following systems of linear equations by Cramer's rule:

$5x + 7y = -2$

$4x + 6y = -3$

Answer

Given: - Two equations $5x + 7y = -2$ and $4x + 6y = -3$

Tip: - Theorem – Cramer's Rule

Let there be a system of n simultaneous linear equations and with n unknown given by

$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$

$\vdots \vdots \vdots$

$a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n1} & \ldots & a_{nn} \end{vmatrix}$$

and let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \ldots, x_n = \frac{D_n}{D}$ provided that $D \neq 0$

Now, here we have

$5x + 7y = -2$

$4x + 6y = -3$

So by comparing with the theorem, let's find D, D_1 and D_2

$$\Rightarrow \ D = \begin{vmatrix} 5 & 7 \\ 4 & 6 \end{vmatrix}$$

Solving determinant, expanding along 1st row

$\Rightarrow D = 5(6) - (7)(4)$

$\Rightarrow D = 30 - 28$

$\Rightarrow D = 2$

Again,

$$\Rightarrow \ D_1 = \begin{vmatrix} -2 & 7 \\ -3 & 6 \end{vmatrix}$$

Solving determinant, expanding along 1st row

$\Rightarrow D_1 = -2(6) - (7)(-3)$

$\Rightarrow D_1 = -12 + 21$

$\Rightarrow D_1 = 9$

and

$$\Rightarrow \ D_2 = \begin{vmatrix} 5 & -2 \\ 4 & -3 \end{vmatrix}$$

Solving determinant, expanding along 1st row

$\Rightarrow D_2 = -3(5) - (-2)(4)$

$\Rightarrow D_2 = -15 + 8$

$\Rightarrow D_2 = -7$

Thus by Cramer's Rule, we have

$\Rightarrow x = \frac{D_1}{D}$

$\Rightarrow x = \frac{9}{2}$

$\Rightarrow x = \frac{9}{2}$

and

$\Rightarrow y = \frac{D_2}{D}$

$\Rightarrow y = \frac{-7}{2}$

$\Rightarrow y = \frac{-7}{2}$

9. Question

Solve the following systems of linear equations by Cramer's rule:

$9x + 5y = 10$

3y – 2x = 8

Answer

Given: - Two equations 9x + 5y = 10 and 3y – 2x = 8

Tip: - Theorem – Cramer’s Rule

Let there be a system of n simultaneous linear equations and with n unknown given by

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

$$\vdots\ \vdots\ \vdots$$

$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n1} & \dots & a_{nn} \end{vmatrix}$$

and let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \dots, x_n = \frac{D_n}{D}$ provided that $D \neq 0$

Now, here we have

9x + 5y = 10

3y – 2x = 8

So by comparing with the theorem, let's find D, D_1 and D_2

$$\Rightarrow D = \begin{vmatrix} 9 & 5 \\ -2 & 3 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} row

⇒ D = 3(9) – (5)(– 2)

⇒ D = 27 + 10

⇒ D = 37

Again,

$$\Rightarrow D_1 = \begin{vmatrix} 10 & 5 \\ 8 & 3 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} row

⇒ D_1 = 10(3) – (8)(5)

⇒ D_1 = 30 – 40

⇒ D_1 = – 10

and

$\Rightarrow D_2 = \begin{vmatrix} 9 & 10 \\ -2 & 8 \end{vmatrix}$

Solving determinant, expanding along 1st row

$\Rightarrow D_2 = 9(8) - (10)(-2)$

$\Rightarrow D_2 = 72 + 20$

$\Rightarrow D_2 = 92$

Thus by Cramer's Rule, we have

$\Rightarrow x = \frac{D_1}{D}$

$\Rightarrow x = \frac{-10}{37}$

$\Rightarrow x = \frac{-10}{37}$

and

$\Rightarrow y = \frac{D_2}{D}$

$\Rightarrow y = \frac{92}{37}$

$\Rightarrow y = \frac{92}{37}$

10. Question

Solve the following systems of linear equations by Cramer's rule:

$x + 2y = 1$

$3x + y = 4$

Answer

Given: - Two equations $x + 2y = 1$ and $3x + y = 4$

Tip: - Theorem – Cramer's Rule

Let there be a system of n simultaneous linear equations and with n unknown given by

$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$

$\vdots \; \vdots \; \vdots$

$a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n$

$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n1} & \ldots & a_{nn} \end{vmatrix}$

and let D_j be the determinant obtained from D after replacing the j^{th} column by

$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$

Then,

$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \ldots, x_n = \frac{D_n}{D}$ provided that $D \neq 0$

Now, here we have

$x + 2y = 1$

$3x + y = 4$

So by comparing with theorem, lets find D, D_1 and D_2

$\Rightarrow D = \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix}$

Solving determinant, expanding along 1st row

$\Rightarrow D = 1(1) - (3)(2)$

$\Rightarrow D = 1 - 6$

$\Rightarrow D = -5$

Again,

$\Rightarrow D_1 = \begin{vmatrix} 1 & 2 \\ 4 & 1 \end{vmatrix}$

Solving determinant, expanding along 1st row

$\Rightarrow D_1 = 1(1) - (2)(4)$

$\Rightarrow D_1 = 1 - 8$

$\Rightarrow D_1 = -7$

and

$\Rightarrow D_2 = \begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix}$

Solving determinant, expanding along 1st row

$\Rightarrow D_2 = 1(4) - (1)(3)$

$\Rightarrow D_2 = 4 - 3$

$\Rightarrow D_2 = 1$

Thus by Cramer's Rule, we have

$\Rightarrow x = \frac{D_1}{D}$

$\Rightarrow x = \frac{-7}{-5}$

$\Rightarrow x = \frac{7}{5}$

and

$\Rightarrow y = \frac{D_2}{D}$

$\Rightarrow y = \frac{1}{-5}$

$\Rightarrow y = -\frac{1}{5}$

11. Question

Solve the following system of the linear equations by Cramer's rule:

$3x + y + z = 2$

$2x - 4y + 3z = -1$

$4x + y - 3z = -11$

Answer

Given: - Equations are: –

$3x + y + z = 2$

$2x - 4y + 3z = -1$

$4x + y - 3z = -11$

Tip: - Theorem – Cramer's Rule

Let there be a system of n simultaneous linear equations and with n unknown given by

$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$

$\vdots \; \vdots \; \vdots$

$a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n1} & \ldots & a_{nn} \end{vmatrix}$$

and let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \ldots, x_n = \frac{D_n}{D}$ provided that $D \neq 0$

Now, here we have

$3x + y + z = 2$

$2x - 4y + 3z = -1$

$4x + y - 3z = -11$

So by comparing with the theorem, let's find D, D_1, D_2 and D_3

$$\Rightarrow D = \begin{vmatrix} 3 & 1 & 1 \\ 2 & -4 & 3 \\ 4 & 1 & -3 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} row

$\Rightarrow D = 3[(-4)(-3) - (3)(1)] - 1[(2)(-3) - 12] + 1[2 - 4(-4)]$

$\Rightarrow D = 3[12 - 3] - [-6 - 12] + [2 + 16]$

$\Rightarrow D = 27 + 18 + 18$

$\Rightarrow D = 63$

Again,

$$\Rightarrow D_1 = \begin{vmatrix} 2 & 1 & 1 \\ -1 & -4 & 3 \\ -11 & 1 & -3 \end{vmatrix}$$

Solving determinant, expanding along 1st row

⇒ D_1 = 2[(– 4)(– 3) – (3)(1)] – 1[(– 1)(– 3) – (– 11)(3)] + 1[(– 1) – (– 4)(– 11)]

⇒ D_1 = 2[12 – 3] – 1[3 + 33] + 1[– 1 – 44]

⇒ D_1 = 2[9] – 36 – 45

⇒ D_1 = 18 – 36 – 45

⇒ D_1 = – 63

Again

$$\Rightarrow D_2 = \begin{vmatrix} 3 & 2 & 1 \\ 2 & -1 & 3 \\ 4 & -11 & -3 \end{vmatrix}$$

Solving determinant, expanding along 1st row

⇒ D_2 = 3[3 + 33] – 2[– 6 – 12] + 1[– 22 + 4]

⇒ D_2 = 3[36] – 2(– 18) – 18

⇒ D_2 = 126

And,

$$\Rightarrow D_3 = \begin{vmatrix} 3 & 1 & 2 \\ 2 & -4 & -1 \\ 4 & 1 & -11 \end{vmatrix}$$

Solving determinant, expanding along 1st row

⇒ D_3 = 3[44 + 1] – 1[– 22 + 4] + 2[2 + 16]

⇒ D_3 = 3[45] – 1(– 18) + 2(18)

⇒ D_3 = 135 + 18 + 36

⇒ D_3 = 189

Thus by Cramer’s Rule, we have

$$\Rightarrow x = \frac{D_1}{D}$$

$$\Rightarrow x = \frac{-63}{63}$$

⇒ x = – 1

again,

$$\Rightarrow y = \frac{D_2}{D}$$

$$\Rightarrow y = \frac{126}{63}$$

⇒ y = 2

and,

$$\Rightarrow z = \frac{D_3}{D}$$

$\Rightarrow z = \frac{189}{63}$

$\Rightarrow z = 3$

12. Question

Solve the following system of the linear equations by Cramer's rule:

x – 4y – z = 11

2x – 5y + 2z = 39

– 3x + 2y + z = 1

Answer

Given: - Equations are: –

x – 4y – z = 11

2x – 5y + 2z = 39

– 3x + 2y + z = 1

Tip: - Theorem – Cramer's Rule

Let there be a system of n simultaneous linear equations and with n unknown given by

$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$

$\vdots \; \vdots \; \vdots$

$a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n1} & \ldots & a_{nn} \end{vmatrix}$$

and let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \ldots, x_n = \frac{D_n}{D}$ provided that D ≠ 0

Now, here we have

x – 4y – z = 11

2x – 5y + 2z = 39

– 3x + 2y + z = 1

So by comparing with theorem, lets find D , D_1 and D_2

$$\Rightarrow D = \begin{vmatrix} 1 & -4 & -1 \\ 2 & -5 & 2 \\ -3 & 2 & 1 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} row

⇒ D = 1[(– 5)(1) – (2)(2)] + 4[(2)(1) + 6] – 1[4 + 5(– 3)]

$\Rightarrow D = 1[- 5 - 4] + 4[8] - [- 11]$

$\Rightarrow D = - 9 + 32 + 11$

$\Rightarrow D = 34$

Again,

$$\Rightarrow D_1 = \begin{vmatrix} 11 & -4 & -1 \\ 39 & -5 & 2 \\ 1 & 2 & 1 \end{vmatrix}$$

Solving determinant, expanding along 1st row

$\Rightarrow D_1 = 11[(- 5)(1) - (2)(2)] + 4[(39)(1) - (2)(1)] - 1[2(39) - (- 5)(1)]$

$\Rightarrow D_1 = 11[- 5 - 4] + 4[39 - 2] - 1[78 + 5]$

$\Rightarrow D_1 = 11[- 9] + 4(37) - 83$

$\Rightarrow D_1 = - 99 - 148 - 45$

$\Rightarrow D_1 = - 34$

Again

$$\Rightarrow D_2 = \begin{vmatrix} 1 & 11 & -1 \\ 2 & 39 & 2 \\ -3 & 1 & 1 \end{vmatrix}$$

Solving determinant, expanding along 1st row

$\Rightarrow D_2 = 1[39 - 2] - 11[2 + 6] - 1[2 + 117]$

$\Rightarrow D_2 = 1[37] - 11(8) - 119$

$\Rightarrow D_2 = - 170$

And,

$$\Rightarrow D_3 = \begin{vmatrix} 1 & -4 & 11 \\ 2 & -5 & 39 \\ -3 & 2 & 1 \end{vmatrix}$$

Solving determinant, expanding along 1st row

$\Rightarrow D_3 = 1[- 5 - (39)(2)] - (- 4)[2 - (39)(- 3)] + 11[4 - (- 5)(- 3)]$

$\Rightarrow D_3 = 1[- 5 - 78] + 4(2 + 117) + 11(4 - 15)$

$\Rightarrow D_3 = - 83 + 4(119) + 11(- 11)$

$\Rightarrow D_3 = 272$

Thus by Cramer's Rule, we have

$$\Rightarrow x = \frac{D_1}{D}$$

$$\Rightarrow x = \frac{-34}{34}$$

$\Rightarrow x = - 1$

again,

$$\Rightarrow y = \frac{D_2}{D}$$

$$\Rightarrow y = \frac{-170}{34}$$

$\Rightarrow y = -5$

and,

$\Rightarrow z = \frac{D_3}{D}$

$\Rightarrow z = \frac{272}{34}$

$\Rightarrow z = 8$

13. Question

Solve the following system of the linear equations by Cramer’s rule:

$6x + y - 3z = 5$

$X + 3y - 2z = 5$

$2x + y + 4z = 8$

Answer

Given: - Equations are: –

$6x + y - 3z = 5$

$X + 3y - 2z = 5$

$2x + y + 4z = 8$

Tip: - Theorem – Cramer’s Rule

Let there be a system of n simultaneous linear equations and with n unknown given by

$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$

$\vdots \; \vdots \; \vdots$

$a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n1} & \ldots & a_{nn} \end{vmatrix}$$

and let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \ldots, x_n = \frac{D_n}{D}$ provided that $D \neq 0$

Now, here we have

$6x + y - 3z = 5$

$x + 3y - 2z = 5$

$2x + y + 4z = 8$

So by comparing with theorem, lets find D , D_1 and D_2

$$\Rightarrow D = \begin{vmatrix} 6 & 1 & -3 \\ 1 & 3 & -2 \\ 2 & 1 & 4 \end{vmatrix}$$

Solving determinant, expanding along 1st Row

⇒ D = 6[(4)(3) – (1)(– 2)] – 1[(4)(1) + 4] – 3[1 – 3(2)]

⇒ D = 6[12 + 2] – [8] – 3[– 5]

⇒ D = 84 – 8 + 15

⇒ D = 91

Again, Solve D_1 formed by replacing 1st column by B matrices

Here

$$B = \begin{vmatrix} 5 \\ 5 \\ 8 \end{vmatrix}$$

$$\Rightarrow D_1 = \begin{vmatrix} 5 & 1 & -3 \\ 5 & 3 & -2 \\ 8 & 1 & 4 \end{vmatrix}$$

Solving determinant, expanding along 1st Row

⇒ D_1 = 5[(4)(3) – (– 2)(1)] – 1[(5)(4) – (– 2)(8)] – 3[(5) – (3)(8)]

⇒ D_1 = 5[12 + 2] – 1[20 + 16] – 3[5 – 24]

⇒ D_1 = 5[14] – 36 – 3(– 19)

⇒ D_1 = 70 – 36 + 57

⇒ D_1 = 91

Again, Solve D_2 formed by replacing 1st column by B matrices

Here

$$B = \begin{vmatrix} 5 \\ 5 \\ 8 \end{vmatrix}$$

$$\Rightarrow D_2 = \begin{vmatrix} 6 & 5 & -3 \\ 1 & 5 & -2 \\ 2 & 8 & 4 \end{vmatrix}$$

Solving determinant

⇒ D_2 = 6[20 + 16] – 5[4 – 2(– 2)] + (– 3)[8 – 10]

⇒ D_2 = 6[36] – 5(8) + (– 3)(– 2)

⇒ D_2 = 182

And, Solve D_3 formed by replacing 1st column by B matrices

Here

$$B = \begin{vmatrix} 5 \\ 5 \\ 8 \end{vmatrix}$$

$$\Rightarrow D_3 = \begin{vmatrix} 6 & 1 & 5 \\ 1 & 3 & 5 \\ 2 & 1 & 8 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} Row

$\Rightarrow D_3 = 6[24 - 5] - 1[8 - 10] + 5[1 - 6]$

$\Rightarrow D_3 = 6[19] - 1(-2) + 5(-5)$

$\Rightarrow D_3 = 114 + 2 - 25$

$\Rightarrow D_3 = 91$

Thus by Cramer's Rule, we have

$\Rightarrow x = \frac{D_1}{D}$

$\Rightarrow x = \frac{91}{91}$

$\Rightarrow x = 1$

again,

$\Rightarrow y = \frac{D_2}{D}$

$\Rightarrow y = \frac{182}{91}$

$\Rightarrow y = 2$

and,

$\Rightarrow z = \frac{D_3}{D}$

$\Rightarrow z = \frac{91}{91}$

$\Rightarrow z = 1$

14. Question

Solve the following system of the linear equations by Cramer's rule:

$x + y = 5$

$y + z = 3$

$x + z = 4$

Answer

Given: - Equations are: –

$x + y = 5$

$y + z = 3$

$x + z = 4$

Tip: - Theorem – Cramer's Rule

Let there be a system of n simultaneous linear equations and with n unknown given by

$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$

$\vdots \; \vdots \; \vdots$

$a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n$

Let D $= \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n1} & \cdots & a_{nn} \end{vmatrix}$

and let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \ldots, x_n = \frac{D_n}{D}$ provided that $D \neq 0$

Now, here we have

x + y = 5

y + z = 3

x + z = 4

So by comparing with theorem, lets find D , D_1 and D_2

$$\Rightarrow D = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}$$

Solving determinant, expanding along 1st Row

⇒ D = 1[1] – 1[– 1] + 0[– 1]

⇒ D = 1 + 1 + 0

⇒ D = 2

⇒ D = 2

Again, Solve D_1 formed by replacing 1st column by B matrices

Here

$$B = \begin{vmatrix} 5 \\ 3 \\ 4 \end{vmatrix}$$

$$\Rightarrow D_1 = \begin{vmatrix} 5 & 1 & 0 \\ 3 & 1 & 1 \\ 4 & 0 & 1 \end{vmatrix}$$

Solving determinant, expanding along 1st Row

⇒ D_1 = 5[1] – 1[(3)(1) – (4)(1)] + 0[0 – (4)(1)]

⇒ D_1 = 5 – 1[3 – 4] + 0[– 4]

⇒ D_1 = 5 – 1[– 1] + 0

⇒ D_1 = 5 + 1 + 0

⇒ D_1 = 6

Again, Solve D_2 formed by replacing 1st column by B matrices

Here

$$B = \begin{vmatrix} 5 \\ 3 \\ 4 \end{vmatrix}$$

$$\Rightarrow D_2 = \begin{vmatrix} 1 & 5 & 0 \\ 0 & 3 & 1 \\ 1 & 4 & 1 \end{vmatrix}$$

Solving determinant

$\Rightarrow D_2 = 1[3 - 4] - 5[-1] + 0[0 - 3]$

$\Rightarrow D_2 = 1[-1] + 5 + 0$

$\Rightarrow D_2 = 4$

And, Solve D_3 formed by replacing 1^{st} column by B matrices

Here

$$B = \begin{vmatrix} 5 \\ 3 \\ 4 \end{vmatrix}$$

$$\Rightarrow D_3 = \begin{vmatrix} 1 & 1 & 5 \\ 0 & 1 & 3 \\ 1 & 0 & 4 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} Row

$\Rightarrow D_3 = 1[4 - 0] - 1[0 - 3] + 5[0 - 1]$

$\Rightarrow D_3 = 1[4] - 1(-3) + 5(-1)$

$\Rightarrow D_3 = 4 + 3 - 5$

$\Rightarrow D_3 = 2$

Thus by Cramer's Rule, we have

$\Rightarrow x = \frac{D_1}{D}$

$\Rightarrow x = \frac{6}{2}$

$\Rightarrow x = 3$

again,

$\Rightarrow y = \frac{D_2}{D}$

$\Rightarrow y = \frac{4}{2}$

$\Rightarrow y = 2$

and,

$\Rightarrow z = \frac{D_3}{D}$

$\Rightarrow z = \frac{2}{2}$

$\Rightarrow z = 1$

15. Question

Solve the following system of the linear equations by Cramer's rule:

2y – 3z = 0

X + 3y = – 4

3x + 4y = 3

Answer

Given: - Equations are: –

2y – 3z = 0

X + 3y = – 4

3x + 4y = 3

Tip: - Theorem – Cramer's Rule

Let there be a system of n simultaneous linear equations and with n unknown given by

$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$

$\vdots \; \vdots \; \vdots$

$a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n$

$$\text{Let D} = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n1} & \cdots & a_{nn} \end{vmatrix}$$

and let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \ldots, x_n = \frac{D_n}{D}$ provided that D ≠ 0

Now, here we have

2y – 3z = 0

x + 3y = – 4

3x + 4y = 3

So by comparing with theorem, lets find D , D_1 and D_2

$$\Rightarrow \; D = \begin{vmatrix} 0 & 2 & -3 \\ 1 & 3 & 0 \\ 3 & 4 & 0 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} Row

⇒ D = 0[0] – 2[(0)(1) – 0] – 3[1(4) – 3(3)]

⇒ D = 0 – 0 – 3[4 – 9]

⇒ D = 0 – 0 + 15

⇒ D = 15

Again, Solve D_1 formed by replacing 1^{st} column by B matrices

Here

$$B = \begin{vmatrix} 0 \\ -4 \\ 3 \end{vmatrix}$$

$$\Rightarrow D_1 = \begin{vmatrix} 0 & 2 & -3 \\ -4 & 3 & 0 \\ 3 & 4 & 0 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} Row

$\Rightarrow D_1 = 0[0] - 2[(0)(-4) - 0] - 3[4(-4) - 3(3)]$

$\Rightarrow D_1 = 0 - 0 - 3[-16 - 9]$

$\Rightarrow D_1 = 0 - 0 - 3(-25)$

$\Rightarrow D_1 = 0 - 0 + 75$

$\Rightarrow D_1 = 75$

Again, Solve D_2 formed by replacing 2^{nd} column by B matrices

Here

$$B = \begin{vmatrix} 0 \\ -4 \\ 3 \end{vmatrix}$$

$$\Rightarrow D_2 = \begin{vmatrix} 0 & 0 & -3 \\ 1 & -4 & 0 \\ 3 & 3 & 0 \end{vmatrix}$$

Solving determinant

$\Rightarrow D_2 = 0[0] - 0[(0)(1) - 0] - 3[1(3) - 3(-4)]$

$\Rightarrow D_2 = 0 - 0 + (-3)(3 + 12)$

$\Rightarrow D_2 = -45$

And, Solve D_3 formed by replacing 3^{rd} column by B matrices

Here

$$B = \begin{vmatrix} 0 \\ -4 \\ 3 \end{vmatrix}$$

$$\Rightarrow D_3 = \begin{vmatrix} 0 & 2 & 0 \\ 1 & 3 & -4 \\ 3 & 4 & 3 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} Row

$\Rightarrow D_3 = 0[9 - (-4)4] - 2[(3)(1) - (-4)(3)] + 0[1(4) - 3(3)]$

$\Rightarrow D_3 = 0[25] - 2(3 + 12) + 0(4 - 9)$

$\Rightarrow D_3 = 0 - 30 + 0$

$\Rightarrow D_3 = -30$

Thus by Cramer's Rule, we have

$$\Rightarrow x = \frac{D_1}{D}$$

$\Rightarrow x = \frac{75}{15}$

⇒ x = 5

again,

$\Rightarrow y = \frac{D_2}{D}$

$\Rightarrow y = \frac{-45}{15}$

⇒ y = - 3

and,

$\Rightarrow z = \frac{D_3}{D}$

$\Rightarrow z = \frac{-30}{15}$

⇒ z = - 2

16. Question

Solve the following system of the linear equations by Cramer's rule:

5x – 7y + z = 11

6x – 8y – z = 15

3x + 2y – 6z = 7

Answer

Given: - Equations are: –

5x – 7y + z = 11

6x – 8y – z = 15

3x + 2y – 6z = 7

Tip: - Theorem – Cramer's Rule

Let there be a system of n simultaneous linear equations and with n unknown given by

$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$

$\vdots \; \vdots \; \vdots$

$a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n$

$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n1} & \ldots & a_{nn} \end{vmatrix}$

and let D_j be the determinant obtained from D after replacing the j^{th} column by

$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$

Then,

$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \ldots, x_n = \frac{D_n}{D}$ provided that $D \neq 0$

Now, here we have

$5x - 7y + z = 11$

$6x - 8y - z = 15$

$3x + 2y - 6z = 7$

So by comparing with theorem, lets find D , D_1 and D_2

$$\Rightarrow D = \begin{vmatrix} 5 & -7 & 1 \\ 6 & -8 & -1 \\ 3 & 2 & -6 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} Row

$\Rightarrow D = 5[(-8)(-6) - (-1)(2)] - 7[(-6)(6) - 3(-1)] + 1[2(6) - 3(-8)]$

$\Rightarrow D = 5[48 + 2] - 7[-36 + 3] + 1[12 + 24]$

$\Rightarrow D = 250 - 231 + 36$

$\Rightarrow D = 55$

Again, Solve D_1 formed by replacing 1^{st} column by B matrices

Here

$$B = \begin{vmatrix} 11 \\ 15 \\ 7 \end{vmatrix}$$

$$\Rightarrow D_1 = \begin{vmatrix} 11 & -7 & 1 \\ 15 & -8 & -1 \\ 7 & 2 & -6 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} Row

$\Rightarrow D_1 = 11[(-8)(-6) - (2)(-1)] - (-7)[(15)(-6) - (-1)(7)] + 1[(15)2 - (7)(-8)]$

$\Rightarrow D_1 = 11[48 + 2] + 7[-90 + 7] + 1[30 + 56]$

$\Rightarrow D_1 = 11[50] + 7[-83] + 86$

$\Rightarrow D_1 = 550 - 581 + 86$

$\Rightarrow D_1 = 55$

Again, Solve D_2 formed by replacing 2^{nd} column by B matrices

Here

$$B = \begin{vmatrix} 11 \\ 15 \\ 7 \end{vmatrix}$$

$$\Rightarrow D_2 = \begin{vmatrix} 5 & 11 & 1 \\ 6 & 15 & -1 \\ 3 & 7 & -6 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} Row

$\Rightarrow D_2 = 5[(15)(-6) - (7)(-1)] - 11[(6)(-6) - (-1)(3)] + 1[(6)7 - (15)(3)]$

$\Rightarrow D_2 = 5[-90 + 7] - 11[-36 + 3] + 1[42 - 45]$

$\Rightarrow D_2 = 5[-83] - 11(-33) - 3$

$\Rightarrow D_2 = -415 + 363 - 3$

$\Rightarrow D_2 = -55$

And, Solve D_3 formed by replacing 3^{rd} column by B matrices

Here

$$B = \begin{vmatrix} 11 \\ 15 \\ 7 \end{vmatrix}$$

$$\Rightarrow D_3 = \begin{vmatrix} 5 & -7 & 11 \\ 6 & -8 & 15 \\ 3 & 2 & 7 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} Row

$\Rightarrow D_3 = 5[(-8)(7) - (15)(2)] - (-7)[(6)(7) - (15)(3)] + 11[(6)2 - (-8)(3)]$

$\Rightarrow D_3 = 5[-56 - 30] - (-7)[42 - 45] + 11[12 + 24]$

$\Rightarrow D_3 = 5[-86] + 7[-3] + 11[36]$

$\Rightarrow D_3 = -430 - 21 + 396$

$\Rightarrow D_3 = -55$

Thus by Cramer's Rule, we have

$$\Rightarrow x = \frac{D_1}{D}$$

$$\Rightarrow x = \frac{55}{55}$$

$\Rightarrow x = 1$

again,

$$\Rightarrow y = \frac{D_2}{D}$$

$$\Rightarrow y = \frac{-55}{55}$$

$\Rightarrow y = -1$

and,

$$\Rightarrow z = \frac{D_3}{D}$$

$$\Rightarrow z = \frac{-55}{55}$$

$\Rightarrow z = -1$

17. Question

Solve the following system of the linear equations by Cramer's rule:

$2x - 3y - 4z = 29$

$-2x + 5y - z = -15$

$3x - y + 5z = -11$

Answer

Given: - Equations are: –

$2x - 3y - 4z = 29$

$-2x + 5y - z = -15$

$3x - y + 5z = -11$

Tip: - Theorem – Cramer's Rule

Let there be a system of n simultaneous linear equations and with n unknown given by

$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$

$\vdots \; \vdots \; \vdots$

$a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n1} & \ldots & a_{nn} \end{vmatrix}$$

and let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \ldots, x_n = \frac{D_n}{D}$ provided that $D \neq 0$

Now, here we have

$2x - 3y - 4z = 29$

$-2x + 5y - z = -15$

$3x - y + 5z = -11$

So by comparing with theorem, lets find D , D_1 and D_2

$$\Rightarrow D = \begin{vmatrix} 2 & -3 & -4 \\ -2 & 5 & -1 \\ 3 & -1 & 5 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} Row

$\Rightarrow D = 2[(5)(5) - (-1)(-1)] - (-3)[(-2)(5) - 3(-1)] + (-4)[(-2)(-1) - 3(5)]$

$\Rightarrow D = 2[25 - 1] + 3[-10 + 3] - 4[2 - 15]$

$\Rightarrow D = 48 - 21 + 52$

$\Rightarrow D = 79$

Again, Solve D_1 formed by replacing 1^{st} column by B matrices

Here

$$B = \begin{vmatrix} 29 \\ -15 \\ -11 \end{vmatrix}$$

$$\Rightarrow D_1 = \begin{vmatrix} 29 & -3 & -4 \\ -15 & 5 & -1 \\ -11 & -1 & 5 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} Row

$\Rightarrow D_1 = 29[(5)(5) - (-1)(-1)] - (-3)[(-15)(5) - (-11)(-1)] + (-4)[(-15)(-1) - (-11)(5)]$

$\Rightarrow D_1 = 29[25 - 1] + 3[-75 - 11] - 4[15 + 55]$

$\Rightarrow D_1 = 29[24] + 3[-86] - 4(70)$

$\Rightarrow D_1 = 696 - 258 - 280$

$\Rightarrow D_1 = 158$

Again, Solve D_2 formed by replacing 2^{nd} column by B matrices

Here

$$B = \begin{vmatrix} 29 \\ -15 \\ -11 \end{vmatrix}$$

$$\Rightarrow D_2 = \begin{vmatrix} 2 & 29 & -4 \\ -2 & -15 & -1 \\ 3 & -11 & 5 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} Row

$\Rightarrow D_2 = 2[(-15)(5) - (-11)(-1)] - 29[(-2)(5) - 3(-1)] + (-4)[(-11)(-2) - 3(-15)]$

$\Rightarrow D_2 = 2[-75 - 11] - 29(-10 + 3) - 4(22 + 45)$

$\Rightarrow D_2 = 2[-86] - 29(-7) - 4(67)$

$\Rightarrow D_2 = -172 + 203 - 268$

$\Rightarrow D_2 = -237$

And, Solve D_3 formed by replacing 1^{st} column by B matrices

Here

$$B = \begin{vmatrix} 29 \\ -15 \\ -11 \end{vmatrix}$$

$$\Rightarrow D_3 = \begin{vmatrix} 2 & -3 & 29 \\ -2 & 5 & -15 \\ 3 & -1 & -11 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} Row

$\Rightarrow D_3 = 2[(5)(-11) - (-15)(-1)] - (-3)[(-11)(-2) - (-15)(3)] + 29[(-2)(-1) - (3)(5)]$

$\Rightarrow D_3 = 2[-55 - 15] + 3(22 + 45) + 29(2 - 15)$

$\Rightarrow D_3 = 2[-70] + 3[67] + 29[-13]$

$\Rightarrow D_3 = -140 + 201 - 377$

$\Rightarrow D_3 = -316$

Thus by Cramer’s Rule, we have

$$\Rightarrow x = \frac{D_1}{D}$$

$$\Rightarrow x = \frac{158}{79}$$

$\Rightarrow x = 2$

again,

$\Rightarrow y = \frac{D_2}{D}$

$\Rightarrow y = \frac{-237}{79}$

⇒ y = – 3

and,

$\Rightarrow z = \frac{D_3}{D}$

$\Rightarrow z = \frac{-316}{79}$

⇒ z = – 4

18. Question

Solve the following system of the linear equations by Cramer's rule:

x + y = 1

x + z = – 6

x – y – 2z = 3

Answer

Given: - Equations are: –

x + y = 1

x + z = – 6

x – y – 2z = 3

Tip: - Theorem – Cramer's Rule

Let there be a system of n simultaneous linear equations and with n unknown given by

$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$

$\vdots \vdots \vdots$

$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n1} & \dots & a_{nn} \end{vmatrix}$$

and let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \dots, x_n = \frac{D_n}{D}$ provided that D ≠ 0

Now, here we have

x + y = 1

x + z = – 6

x – y – 2z = 3

So by comparing with theorem, lets find D , D_1 and D_2

$$\Rightarrow\ D = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & -2 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} Row

⇒ D = 1[(0)(– 2) – (1)(– 1)] – 1[(– 2)(1) – 1] + 0[– 1 – 0]

⇒ D = 1[0 + 1] – 1[– 3] – 0[– 2]

⇒ D = 1 + 3 + 0

⇒ D = 4

Again, Solve D_1 formed by replacing 1^{st} column by B matrices

Here

$$B = \begin{vmatrix} 1 \\ -6 \\ 3 \end{vmatrix}$$

$$\Rightarrow\ D_1 = \begin{vmatrix} 1 & 1 & 0 \\ -6 & 0 & 1 \\ 3 & -1 & -2 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} Row

⇒ D_1 = 1[(0)(– 2) – (1)(– 1)] – 1[(– 2)(– 6) – 3] + 0[6 – 0]

⇒ D_1 = 1[0 + 1] – 1[12 – 3] + 0[6]

⇒ D_1 = 1[1] – 9 + 0

⇒ D_1 = 1 – 9 + 0

⇒ D_1 = – 8

Again, Solve D_2 formed by replacing 2^{nd} column by B matrices

Here

$$B = \begin{vmatrix} 1 \\ -6 \\ 3 \end{vmatrix}$$

$$\Rightarrow\ D_2 = \begin{vmatrix} 1 & 1 & 0 \\ 1 & -6 & 1 \\ 1 & 3 & -2 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} Row

⇒ D_2 = 1[(– 6)(– 2) – (1)(3)] – 1[(– 2)(1) – 1] + 0[3 + 6]

⇒ D_2 = 1[12 – 3] – 1(– 2 – 1) + 0(9)

⇒ D_2 = 9 + 3

⇒ D_2 = 12

And, Solve D_3 formed by replacing 3^{rd} column by B matrices

Here

$$B = \begin{vmatrix} 1 \\ -6 \\ 3 \end{vmatrix}$$

$$\Rightarrow D_3 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & -6 \\ 1 & -1 & 3 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} Row

$\Rightarrow D_3 = 1[(0)(3) - (-1)(-6)] - 1[(3)(1) - 1(-6)] + 1[-1 + 0]$

$\Rightarrow D_3 = 1[0 - 6] - 1(3 + 6) + 1(-1)$

$\Rightarrow D_3 = -6 - 9 - 1$

$\Rightarrow D_3 = -16$

Thus by Cramer's Rule, we have

$\Rightarrow x = \frac{D_1}{D}$

$\Rightarrow x = \frac{-8}{4}$

$\Rightarrow x = -2$

again,

$\Rightarrow y = \frac{D_2}{D}$

$\Rightarrow y = \frac{12}{4}$

$\Rightarrow y = 3$

and,

$\Rightarrow z = \frac{D_3}{D}$

$\Rightarrow z = \frac{-16}{4}$

$\Rightarrow z = -4$

19. Question

Solve the following system of the linear equations by Cramer's rule:

$x + y + z + 1 = 0$

$ax + by + cz + d = 0$

$a^2x + b^2y + c^2z + d^2 = 0$

Answer

Given: - Equations are: –

$x + y + z + 1 = 0$

$ax + by + cz + d = 0$

$a^2x + b^2y + c^2z + d^2 = 0$

Tip: - Theorem – Cramer's Rule

Let there be a system of n simultaneous linear equations and with n unknown given by

$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$

$\vdots \vdots \vdots$

$a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n$

Let $D = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n1} & \cdots & a_{nn} \end{vmatrix}$

and let D_j be the determinant obtained from D after replacing the j^{th} column by

$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$

Then,

$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \ldots, x_n = \frac{D_n}{D}$ provided that $D \neq 0$

Now, here we have

$x + y + z + 1 = 0$

$ax + by + cz + d = 0$

$a^2x + b^2y + c^2z + d^2 = 0$

So by comparing with theorem, lets find D , D_1 , D_2 and D_3

$\Rightarrow D = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$

applying, $c_2 \rightarrow c_2 - c_1, c_3 \rightarrow c_3 - c_1$

$\Rightarrow D = \begin{vmatrix} 1 & 0 & 0 \\ a & b - a & c - a \\ a^2 & b^2 - a^2 & c^2 - a^2 \end{vmatrix}$

Take (b – a) from c_2 , and (c – a) from c_3 common, we get

$\Rightarrow D = (b - a)(c - a) \begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^2 & b + a & c + a \end{vmatrix}$

Solving determinant, expanding along 1^{st} Row

$\Rightarrow D = (b - a)(c - a)1[c + a - (b + a)]$

$\Rightarrow D = (b - a)(c - a)(c + a - b - a)$

$\Rightarrow D = (b - a)(c - a)(c - b)$

$\Rightarrow D = (a - b)(b - c)(c - a)$

Again, Solve D_1 formed by replacing 1^{st} column by B matrices

Here

$B = \begin{vmatrix} -1 \\ -d \\ -d^2 \end{vmatrix}$

$$\Rightarrow \ D_1 = \begin{vmatrix} -1 & 1 & 1 \\ -d & b & c \\ -d^2 & b^2 & c^2 \end{vmatrix}$$

applying, $c_2 \rightarrow c_2 - c_1, c_3 \rightarrow c_3 - c_1$

$$\Rightarrow D_1 = -\begin{vmatrix} 1 & 0 & 0 \\ d & b-d & c-d \\ d^2 & b^2-d^2 & c^2-d^2 \end{vmatrix}$$

Take (b – d) from c_2 , and (c – d) from c_3 common, we get

$$\Rightarrow D_1 = -(b-d)(c-d)\begin{vmatrix} 1 & 0 & 0 \\ d & 1 & 1 \\ d^2 & b+d & c+d \end{vmatrix}$$

Solving determinant, expanding along 1st Row

$\Rightarrow D_1 = -(b-d)(c-d)1[c+d-(b+d)]$

$\Rightarrow D_1 = -(b-d)(c-d)(c+d-b-d)$

$\Rightarrow D_1 = -(b-d)(c-d)(c-b)$

$\Rightarrow D_1 = -(d-b)(b-c)(c-d)$

Again, Solve D_2 formed by replacing 2nd column by B matrices

Here

$$B = \begin{vmatrix} -1 \\ -d \\ -d^2 \end{vmatrix}$$

$$\Rightarrow \ D_2 = \begin{vmatrix} 1 & -1 & 1 \\ a & -d & c \\ a^2 & -d^2 & c^2 \end{vmatrix}$$

applying, $c_2 \rightarrow c_2 - c_1, c_3 \rightarrow c_3 - c_1$

$$\Rightarrow D_2 = -\begin{vmatrix} 1 & 0 & 0 \\ a & d-a & c-a \\ a^2 & d^2-a^2 & c^2-a^2 \end{vmatrix}$$

Take (d – a) from c_2 , and (c – a) from c_3 common, we get

$$\Rightarrow D_2 = -(d-a)(c-a)\begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^2 & d+a & c+a \end{vmatrix}$$

Solving determinant, expanding along 1st Row

$\Rightarrow D_2 = -(d-a)(c-a)1[c+a-(d+a)]$

$\Rightarrow D_2 = -(d-a)(c-a)(c+a-d-a)$

$\Rightarrow D_2 = -(d-a)(c-a)(c-d)$

$\Rightarrow D_2 = -(a-d)(d-c)(c-a)$

And, Solve D_3 formed by replacing 3rd column by B matrices

Here

$$B = \begin{vmatrix} -1 \\ -d \\ -d^2 \end{vmatrix}$$

$$\Rightarrow D_3 = \begin{vmatrix} 1 & 1 & -1 \\ a & b & -d \\ a^2 & b^2 & -d^2 \end{vmatrix}$$

applying, $c_2 \to c_2 - c_1, c_3 \to c_3 - c_1$

$$\Rightarrow D_3 = -\begin{vmatrix} 1 & 0 & 0 \\ a & b-a & d-a \\ a^2 & b^2-a^2 & d^2-d^2 \end{vmatrix}$$

Take (b – a) from c_2 , and (d – a) from c_3 common, we get

$$\Rightarrow D_3 = -(b-a)(d-a)\begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^2 & b+a & d+a \end{vmatrix}$$

Solving determinant, expanding along 1st Row

$\Rightarrow D_3 = -(b - d)(c - d)1[a + d - (b + a)]$

$\Rightarrow D_3 = -(b - d)(c - d)(a + d - b - a)$

$\Rightarrow D_3 = -(b - d)(c - d)(d - b)$

$\Rightarrow D_3 = -(d - b)(b - d)(c - d)$

Thus by Cramer's Rule, we have

$$\Rightarrow x = \frac{D_1}{D}$$

$$\Rightarrow x = -\frac{(b-c)(c-d)(d-b)}{(a-b)(b-c)(c-a)}$$

again,

$$\Rightarrow y = \frac{D_2}{D}$$

$$\Rightarrow y = -\frac{(a-d)(d-c)(c-a)}{(a-b)(b-c)(c-a)}$$

and,

$$\Rightarrow z = \frac{D_3}{D}$$

$$\Rightarrow z = -\frac{(a-b)(b-d)(d-a)}{(a-b)(b-c)(c-a)}$$

20. Question

Solve the following system of the linear equations by Cramer's rule:

$x + y + z + w = 2$

$x - 2y + 2z + 2w = -6$

$2x + y - 2z + 2w = -5$

$3x - y + 3z - 3w = -3$

Answer

Given: - Equations are: –

$x + y + z + w = 2$

$x - 2y + 2z + 2w = -6$

$2x + y - 2z + 2w = -5$

$3x - y + 3z - 3w = -3$

Tip: - Theorem – Cramer’s Rule

Let there be a system of n simultaneous linear equations and with n unknown given by

$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$

$\vdots \vdots \vdots$

$a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n1} & \ldots & a_{nn} \end{vmatrix}$$

and let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \ldots, x_n = \frac{D_n}{D}$ provided that $D \neq 0$

Now, here we have

$x + y + z + w = 2$

$x - 2y + 2z + 2w = -6$

$2x + y - 2z + 2w = -5$

$3x - y + 3z - 3w = -3$

So by comparing with theorem, lets find D, D_1, D_2,D_3 and D_4

$$\Rightarrow D = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -2 & 2 & 2 \\ 2 & 1 & -2 & 2 \\ 3 & -1 & 3 & -3 \end{vmatrix}$$

applying, $c_2 \to c_2 - c_1, c_3 \to c_3 - c_1, c_4 \to c_4 - c_1$

$$\Rightarrow D = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & -3 & 1 & 1 \\ 2 & -1 & -4 & 0 \\ 3 & -4 & 0 & -6 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} Row

$$\Rightarrow D = 1\begin{vmatrix} -3 & 1 & 1 \\ -1 & -4 & 0 \\ -4 & 0 & -6 \end{vmatrix}$$

applying, $c_1 \to c_1 + 3c_3, c_2 \to c_2 - c_3$

$$\Rightarrow D = 1\begin{vmatrix} 0 & 0 & 1 \\ -1 & -4 & 0 \\ -22 & 6 & -6 \end{vmatrix}$$

$\Rightarrow D = 1[-6 - 88]$

$\Rightarrow$ D = – 94

Again, Solve D_1 formed by replacing 1st column by B matrices

Here

$$B = \begin{vmatrix} 2 \\ -6 \\ -5 \\ -3 \end{vmatrix}$$

$$\Rightarrow \quad D_1 = \begin{vmatrix} 2 & 1 & 1 & 1 \\ -6 & -2 & 2 & 2 \\ -5 & 1 & -2 & 2 \\ -3 & -1 & 3 & -3 \end{vmatrix}$$

applying, $c_1 \rightarrow c_1 - 2c_4, c_2 \rightarrow c_2 - c_4, c_3 \rightarrow c_3 - c_4$

$$\Rightarrow D_1 = \begin{vmatrix} 0 & 0 & 0 & 1 \\ -10 & -4 & 0 & 2 \\ -9 & -1 & -4 & 2 \\ 3 & 2 & 6 & -3 \end{vmatrix}$$

Solving determinant, expanding along 1st Row

$$\Rightarrow D_1 = -1 \begin{vmatrix} -10 & -4 & 0 \\ -9 & -1 & -4 \\ 3 & 2 & 6 \end{vmatrix}$$

$\Rightarrow D_1 = -1\{(-10)[6(-1) - 2(-4)] - (-4)[(-9)6 - (-4)3] + 0\}$

$\Rightarrow D_1 = -1\{-10[-6 + 8] + 4[-54 + 12]\}$

$\Rightarrow D_1 = -1\{-10[2] + 4[-42]\}$

$\Rightarrow D_1 = 188$

Again, Solve D_2 formed by replacing 2nd column by B matrices

Here

$$B = \begin{vmatrix} 2 \\ -6 \\ -5 \\ -3 \end{vmatrix}$$

$$\Rightarrow \quad D_2 = \begin{vmatrix} 1 & 2 & 1 & 1 \\ 1 & -6 & 2 & 2 \\ 2 & -5 & -2 & 2 \\ 3 & -3 & 3 & -3 \end{vmatrix}$$

applying, $c_1 \rightarrow c_1 - c_4, c_2 \rightarrow c_2 - 2c_4, c_3 \rightarrow c_3 - c_4$

$$\Rightarrow D_2 = \begin{vmatrix} 0 & 0 & 0 & 1 \\ -1 & -10 & 0 & 2 \\ 0 & -9 & -4 & 2 \\ 6 & 3 & 6 & -3 \end{vmatrix}$$

Solving determinant, expanding along 1st Row

$$\Rightarrow D_2 = -1 \begin{vmatrix} -1 & -10 & 0 \\ 0 & -9 & -4 \\ 6 & 3 & 6 \end{vmatrix}$$

$\Rightarrow D_2 = -1\{(-1)[6(-9) - 3(-4)] - (-10)[0 - 6(-4)] + 0[0 + 54]\}$

$\Rightarrow D_2 = -1\{-1[-54 + 12] + 10(24) + 0\}$

$\Rightarrow D_2 = -282$

Again, Solve D_3 formed by replacing 3^{rd} column by B matrices

Here

$$B = \begin{vmatrix} 2 \\ -6 \\ -5 \\ -3 \end{vmatrix}$$

$$\Rightarrow D_3 = \begin{vmatrix} 1 & 1 & 2 & 1 \\ 1 & -2 & -6 & 2 \\ 2 & 1 & -5 & 2 \\ 3 & -1 & -3 & -3 \end{vmatrix}$$

applying, $c_1 \rightarrow c_1 - c_4, c_2 \rightarrow c_2 - c_4, c_3 \rightarrow c_3 - 2c_4$

$$\Rightarrow D_3 = \begin{vmatrix} 0 & 0 & 0 & 1 \\ -1 & -4 & -10 & 2 \\ 0 & -1 & -9 & 2 \\ 6 & 2 & 3 & -3 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} Row

$$\Rightarrow D_3 = -1\begin{vmatrix} -1 & -4 & -10 \\ 0 & -1 & -9 \\ 6 & 2 & 3 \end{vmatrix}$$

$\Rightarrow D_3 = -1\{(-1)[-3-(-9)2] - (-4)[0-6(-9)] + (-10)[0+6]\}$

$\Rightarrow D_3 = -1\{-1[15] + 4(54) - 10(6)\}$

$\Rightarrow D_3 = -1\{-15 + 216 - 60\}$

$\Rightarrow D_3 = -141$

And, Solve D_4 formed by replacing 4^{th} column by B matrices

Here

$$B = \begin{vmatrix} 2 \\ -6 \\ -5 \\ -3 \end{vmatrix}$$

$$\Rightarrow D_4 = \begin{vmatrix} 1 & 1 & 1 & 2 \\ 1 & -2 & 2 & -6 \\ 2 & 1 & -2 & -5 \\ 3 & -1 & 3 & -3 \end{vmatrix}$$

applying, $c_2 \rightarrow c_2 - c_1, c_3 \rightarrow c_3 - c_1, c_4 \rightarrow c_4 - 2c_1$

$$\Rightarrow D_4 = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & -3 & 1 & -8 \\ 2 & -1 & -4 & -9 \\ 3 & -4 & 0 & -9 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} Row

$$\Rightarrow D_4 = 1\begin{vmatrix} -3 & 1 & -8 \\ -1 & -4 & -9 \\ -4 & 0 & -9 \end{vmatrix}$$

$\Rightarrow D_4 = (-3)[(-9)(-4) - 0] - 1[9 - (-4)(-9)] + (-8)[0 - 16]$

$\Rightarrow D_4 = -3[36] - 1(9 - 36) - 8(-16)$

$\Rightarrow D_4 = -108 + 27 + 128$

$\Rightarrow D_4 = 47$

Thus by Cramer's Rule, we have

$\Rightarrow x = \frac{D_1}{D}$

$\Rightarrow x = \frac{188}{-94}$

$\Rightarrow x = -2$

again,

$\Rightarrow y = \frac{D_2}{D}$

$\Rightarrow y = \frac{-282}{-94}$

$\Rightarrow y = 3$

again,

$\Rightarrow z = \frac{D_3}{D}$

$\Rightarrow z = \frac{-141}{-94}$

$\Rightarrow z = \frac{3}{2}$

And,

$\Rightarrow w = \frac{D_4}{D}$

$\Rightarrow w = \frac{47}{-94}$

$\Rightarrow w = -\frac{1}{2}$

21. Question

Solve the following system of the linear equations by Cramer's rule:

$2x - 3z + w = 1$

$x - y + 2w = 1$

$-3y + z + w = 1$

$x + y + z = 1$

Answer

Given: - Equations are: –

$2x - 3z + w = 1$

$x - y + 2w = 1$

$-3y + z + w = 1$

$x + y + z = 1$

Tip: - Theorem – Cramer's Rule

Let there be a system of n simultaneous linear equations and with n unknown given by

$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$

$\vdots \vdots \vdots$

$a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n$

Let $D = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n1} & \cdots & a_{nn} \end{vmatrix}$

and let D_j be the determinant obtained from D after replacing the j^{th} column by

$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$

Then,

$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \ldots, x_n = \frac{D_n}{D}$ provided that $D \neq 0$

Now, here we have

$2x - 3z + w = 1$

$x - y + 2w = 1$

$-3y + z + w = 1$

$x + y + z = 1$

So by comparing with theorem, lets find D, D_1, D_2,D_3 and D_4

$$\Rightarrow D = \begin{vmatrix} 2 & 0 & -3 & 1 \\ 1 & -1 & 0 & 2 \\ 0 & -3 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix}$$

applying, $c_2 \rightarrow c_2 - c_1, c_3 \rightarrow c_3 - c_1$

$$\Rightarrow D = \begin{vmatrix} 2 & -2 & -5 & 1 \\ 1 & -2 & -1 & 2 \\ 0 & -3 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{vmatrix}$$

Solving determinant, expanding along 4^{th} Row

$$\Rightarrow D = -1\begin{vmatrix} -2 & -5 & 1 \\ -2 & -1 & 2 \\ -3 & 1 & 1 \end{vmatrix}$$

applying, $c_1 \rightarrow c_1 + 3c_3, c_2 \rightarrow c_2 - c_3$

$$\Rightarrow D = 1\begin{vmatrix} 1 & -6 & 1 \\ 4 & -3 & 2 \\ 0 & 0 & 1 \end{vmatrix}$$

expanding along 3^{rd} row

$\Rightarrow D = -1[-3 - (-6)4]$

$\Rightarrow D = -21$

Again, Solve D_1 formed by replacing 1^{st} column by B matrices

Here

$$B = \begin{vmatrix} 1 \\ 1 \\ 1 \\ 1 \end{vmatrix}$$

$$\Rightarrow \ D_1 = \begin{vmatrix} 1 & 0 & -3 & 1 \\ 1 & -1 & 0 & 2 \\ 1 & -3 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix}$$

applying, $c_3 \to c_3 + 3c_1, c_4 \to c_4 - c_1$

$$\Rightarrow D_1 = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & -1 & 3 & 1 \\ 1 & -3 & 4 & 0 \\ 1 & 1 & 4 & -1 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} Row

$$\Rightarrow D_1 = 1 \begin{vmatrix} -1 & 3 & 1 \\ -3 & 4 & 0 \\ 1 & 4 & -1 \end{vmatrix}$$

$\Rightarrow D_1 = (-1)[(4)(-1) - 0(4)] - (3)[(-3)(-1) - 0] + 1[-12 - 4]$

$\Rightarrow D_1 = -1[-4 - 0] - 3[3 - 0] - 16$

$\Rightarrow D_1 = 4 - 9 - 16$

$\Rightarrow D_1 = -21$

Again, Solve D_2 formed by replacing 2^{nd} column by B matrices

Here

$$B = \begin{vmatrix} 1 \\ 1 \\ 1 \\ 1 \end{vmatrix}$$

$$\Rightarrow \ D_2 = \begin{vmatrix} 2 & 1 & -3 & 1 \\ 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix}$$

applying, $c_2 \to c_2 - c_1, c_3 \to c_3 - c_1$

$$\Rightarrow D_2 = \begin{vmatrix} 2 & -1 & -5 & 1 \\ 1 & 0 & -1 & 2 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{vmatrix}$$

Solving determinant, expanding along 4^{th} Row

$$\Rightarrow D_2 = -1 \begin{vmatrix} -1 & -5 & 1 \\ 0 & -1 & 2 \\ 1 & 1 & 1 \end{vmatrix}$$

$\Rightarrow D_2 = -1\{(-1)[1(-1) - 1(2)] - (-5)[0 - 1(2)] + 1[0 - (-1)]\}$

$\Rightarrow D_2 = -1\{-1[-1 - 2] + 5(-2) + 1\}$

$\Rightarrow D_2 = 6$

Again, Solve D_3 formed by replacing 3^{rd} column by B matrices

Here

$$B = \begin{vmatrix} 1 \\ 1 \\ 1 \\ 1 \end{vmatrix}$$

$$\Rightarrow D_3 = \begin{vmatrix} 2 & 0 & 1 & 1 \\ 1 & -1 & 1 & 2 \\ 0 & -3 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix}$$

applying, $c_2 \to c_2 - c_1, c_3 \to c_3 - c_1$

$$\Rightarrow D_3 = \begin{vmatrix} 2 & -2 & -1 & 1 \\ 1 & -2 & 0 & 2 \\ 0 & -3 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{vmatrix}$$

Solving determinant, expanding along 4th Row

$$\Rightarrow D_3 = -1 \begin{vmatrix} -2 & -1 & 1 \\ -2 & 0 & 2 \\ -3 & 1 & 1 \end{vmatrix}$$

$\Rightarrow D_3 = -1\{(-2)[0-(1)2]-(-1)[-2-(-3)(2)]+1[-2-0]\}$

$\Rightarrow D_3 = -1\{-2[-2]+1(-2+6)+1(-2)\}$

$\Rightarrow D_3 = -1\{4+4-2\}$

$\Rightarrow D_3 = -6$

And, Solve D_4 formed by replacing 4th column by B matrices

Here

$$B = \begin{vmatrix} 1 \\ 1 \\ 1 \\ 1 \end{vmatrix}$$

$$\Rightarrow D_4 = \begin{vmatrix} 2 & 0 & -3 & 1 \\ 1 & -1 & 0 & 1 \\ 0 & -3 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{vmatrix}$$

applying, $c_2 \to c_2 - c_1, c_3 \to c_3 - c_1, c_4 \to c_4 - c_1$

$$\Rightarrow D_4 = \begin{vmatrix} 2 & -2 & -5 & -1 \\ 1 & -2 & -1 & 0 \\ 0 & -3 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{vmatrix}$$

Solving determinant, expanding along 4th Row

$$\Rightarrow D_4 = -1 \begin{vmatrix} -2 & -5 & -1 \\ -2 & -1 & 0 \\ -3 & 1 & 1 \end{vmatrix}$$

$\Rightarrow D_4 = (-1)\{(-2)[(-1)1-0]-(-5)[-2-0]+(-1)[-2-3]\}$

$\Rightarrow D_4 = (-1)\{2-10+5\}$

$\Rightarrow D_4 = 3$

$\Rightarrow D_4 = 3$

Thus by Cramer’s Rule, we have

$\Rightarrow x = \frac{D_1}{D}$

$\Rightarrow x = \frac{-21}{-21}$

$\Rightarrow x = 1$

again,

$\Rightarrow y = \frac{D_2}{D}$

$\Rightarrow y = \frac{6}{-21}$

$\Rightarrow y = -\frac{2}{7}$

again,

$\Rightarrow z = \frac{D_3}{D}$

$\Rightarrow z = \frac{-6}{-21}$

$\Rightarrow z = \frac{2}{7}$

And,

$\Rightarrow w = \frac{D_4}{D}$

$\Rightarrow w = \frac{3}{-21}$

$\Rightarrow w = -\frac{1}{7}$

22. Question

Show that each of the following systems of linear equations is inconsistent:

2x – y = 5

4x – 2y = 7

Answer

Given: - Two equation 2x – y = 5 and 4x – 2y = 7

Tip: - We know that

For a system of 2 simultaneous linear equation with 2 unknowns

(i) If D ≠ 0, then the given system of equations is consistent and has a unique solution given by

$x = \frac{D_1}{D}, y = \frac{D_2}{D}$

(ii) If D = 0 and $D_1 = D_2 = 0$, then the system is consistent and has infinitely many solution.

(iii) If D = 0 and one of D_1 and D_2 is non – zero, then the system is inconsistent.

Now,

We have,

2x – y = 5

4x – 2y = 7

Lets find D

$\Rightarrow D = \begin{vmatrix} 2 & -1 \\ 4 & -2 \end{vmatrix}$

$\Rightarrow$ D = – 4 + 4

$\Rightarrow$ D = 0

Again, D_1 by replacing 1st column by B

Here

$B = \begin{vmatrix} 5 \\ 7 \end{vmatrix}$

$\Rightarrow D_1 = \begin{vmatrix} 5 & -1 \\ 7 & -2 \end{vmatrix}$

$\Rightarrow D_1$ = – 10 + 7

$\Rightarrow D_1$ = – 3

And, D_2 by replacing 2nd column by B

Here

$B = \begin{vmatrix} 5 \\ 7 \end{vmatrix}$

$\Rightarrow D_2 = \begin{vmatrix} 2 & 5 \\ 4 & 7 \end{vmatrix}$

$\Rightarrow D_2$ = 14 – 20

$\Rightarrow D_2$ = – 6

So, here we can see that

D = 0 and D_1 and D_2 are non – zero

Hence the given system of equation is inconsistent.

23. Question

Show that each of the following systems of linear equations is inconsistent:

3x + y = 5

– 6x – 2y = 9

Answer

Given: - Two equation 3x + y = 5 and – 6x – 2y = 9

Tip: - We know that

For a system of 2 simultaneous linear equation with 2 unknowns

(i) If D ≠ 0, then the given system of equations is consistent and has a unique solution given by

$x = \frac{D_1}{D}, y = \frac{D_2}{D}$

(ii) If D = 0 and $D_1 = D_2 = 0$, then the system is consistent and has infinitely many solution.

(iii) If D = 0 and one of D_1 and D_2 is non – zero, then the system is inconsistent.

Now,

We have,

3x + y = 5

$-6x - 2y = 9$

Lets find D

$\Rightarrow D = \begin{vmatrix} 3 & 1 \\ -6 & -2 \end{vmatrix}$

$\Rightarrow D = -6 - 6$

$\Rightarrow D = 0$

Again, D_1 by replacing 1^{st} column by B

Here

$B = \begin{vmatrix} 5 \\ 9 \end{vmatrix}$

$\Rightarrow D_1 = \begin{vmatrix} 5 & 1 \\ 9 & -2 \end{vmatrix}$

$\Rightarrow D_1 = -10 - 9$

$\Rightarrow D_1 = -19$

And, D_2 by replacing 2^{nd} column by B

Here

$B = \begin{vmatrix} 5 \\ 9 \end{vmatrix}$

$\Rightarrow D_2 = \begin{vmatrix} 3 & 5 \\ -6 & 9 \end{vmatrix}$

$\Rightarrow D_2 = 27 + 30$

$\Rightarrow D_2 = 57$

So, here we can see that

$D = 0$ and D_1 and D_2 are non – zero

Hence the given system of equation is inconsistent.

24. Question

Show that each of the following systems of linear equations is inconsistent:

$3x - y + 2z = 3$

$2x + y + 3z = 5$

$x - 2y - z = 1$

Answer

Given: - Three equation

$3x - y + 2z = 3$

$2x + y + 3z = 5$

$x - 2y - z = 1$

Tip: - We know that

For a system of 3 simultaneous linear equation with 3 unknowns

(i) If $D \neq 0$, then the given system of equations is consistent and has a unique solution given by

$$x = \frac{D_1}{D}, y = \frac{D_2}{D} \text{ and } z = \frac{D_3}{D}$$

(ii) If D = 0 and $D_1 = D_2 = D_3 = 0$, then the given system of equation may or may not be consistent. However if consistent, then it has infinitely many solutions.

(iii) If D = 0 and at least one of the determinants D_1, D_2 and D_3 is non – zero, then the system is inconsistent.

Now,

We have,

3x – y + 2z = 3

2x + y + 3z = 5

x – 2y – z = 1

Lets find D

$$\Rightarrow D = \begin{vmatrix} 3 & -1 & 2 \\ 2 & 1 & 3 \\ 1 & -2 & -1 \end{vmatrix}$$

Expanding along 1st row

⇒ D = 3[– 1 – 3(– 2)] – (– 1)[(– 1)2 – 3] + 2[– 4 – 1]

⇒ D = 3[5] + 1[– 5] + 2[– 5]

⇒ D = 0

Again, D_1 by replacing 1st column by B

Here

$$B = \begin{vmatrix} 3 \\ 5 \\ 1 \end{vmatrix}$$

$$\Rightarrow D_1 = \begin{vmatrix} 3 & -1 & 2 \\ 5 & 1 & 3 \\ 1 & -2 & -1 \end{vmatrix}$$

⇒ D_1 = 3[– 1 – 3(– 2)] – (– 1)[(– 1)5 – 3] + 2[– 10 – 1]

⇒ D_1 = 3[5] + [– 8] + 2[– 11]

⇒ D_1 = 15 – 8 – 22

⇒ D_1 = – 15

⇒ $D_1 \neq 0$

So, here we can see that

D = 0 and D_1 is non – zero

Hence the given system of equation is inconsistent.

Hence Proved

25. Question

Show that each of the following systems of linear equations is inconsistent:

x + y + z = 3

2x – y + z = 2

3x + 6y + 5z = 20.

Answer

Given: - Three equation

$x + y + z = 3$

$2x - y + z = 2$

$3x + 6y + 5z = 20.$

Tip: - We know that

For a system of 3 simultaneous linear equation with 3 unknowns

(i) If $D \neq 0$, then the given system of equations is consistent and has a unique solution given by

$$x = \frac{D_1}{D}, y = \frac{D_2}{D} \text{ and } z = \frac{D_3}{D}$$

(ii) If $D = 0$ and $D_1 = D_2 = D_3 = 0$, then the given system of equation may or may not be consistent. However if consistent, then it has infinitely many solution.

(iii) If $D = 0$ and at least one of the determinants D_1, D_2 and D_3 is non – zero, then the system is inconsistent.

Now,

We have,

$x + y + z = 3$

$2x - y + z = 2$

$3x + 6y + 5z = 20.$

Lets find D

$$\Rightarrow D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 3 & 6 & 5 \end{vmatrix}$$

Expanding along 1^{st} row

$\Rightarrow D = 1[-5 - 1(6)] - (1)[(5)2 - 3] + 1[12 + 3]$

$\Rightarrow D = 1[-11] - 1[7] + 1[15]$

$\Rightarrow D = -3$

So, here we can see that

$D \neq 0$

Hence the given system of equation is consistent.

26. Question

Show that each of the following systems of linear equations has infinite number of solutions and solve:

$x - y + z = 3$

$2x + y - z = 2$

$-x - 2y + 2z = 1$

Answer

Given: - Three equation

$x - y + z = 3$

$2x + y - z = 2$

$-x - 2y + 2z = 1$

Tip: - We know that

For a system of 3 simultaneous linear equation with 3 unknowns

(i) If $D \neq 0$, then the given system of equations is consistent and has a unique solution given by

$$x = \frac{D_1}{D}, y = \frac{D_2}{D} \text{ and } z = \frac{D_3}{D}$$

(ii) If $D = 0$ and $D_1 = D_2 = D_3 = 0$, then the given system of equation may or may not be consistent. However if consistent, then it has infinitely many solution.

(iii) If $D = 0$ and at least one of the determinants D_1, D_2 and D_3 is non – zero, then the system is inconsistent.

Now,

We have,

$x - y + z = 3$

$2x + y - z = 2$

$-x - 2y + 2z = 1$

Lets find D

$$\Rightarrow D = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ -1 & -2 & 2 \end{vmatrix}$$

Expanding along 1st row

$\Rightarrow D = 1[2 - (-1)(-2)] - (-1)[(2)2 - (1)] + 1[-4 - (-1)]$

$\Rightarrow D = 1[0] + 1[3] + [-3]$

$\Rightarrow D = 0$

Again, D_1 by replacing 1st column by B

Here

$$B = \begin{vmatrix} 3 \\ 2 \\ 1 \end{vmatrix}$$

$$\Rightarrow D_1 = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 1 & -1 \\ 1 & -2 & 2 \end{vmatrix}$$

$\Rightarrow D_1 = 3[2 - (-1)(-2)] - (-1)[(2)2 - (-1)] + 1[-4 - 1]$

$\Rightarrow D_1 = 3[2 - 2] + [4 + 1] + 1[-5]$

$\Rightarrow D_1 = 0 + 5 - 5$

$\Rightarrow D_1 = 0$

Also, D_2 by replacing 2nd column by B

Here

$$B = \begin{vmatrix} 3 \\ 2 \\ 1 \end{vmatrix}$$

$$\Rightarrow D_2 = \begin{vmatrix} 1 & 3 & 1 \\ 2 & 2 & -1 \\ -1 & 1 & 2 \end{vmatrix}$$

$\Rightarrow D_2 = 1[4 - (-1)(1)] - (3)[(2)2 - (1)] + 1[2 - (-2)]$

$\Rightarrow D_2 = 1[4 + 1] - 3[4 - 1] + 1[4]$

$\Rightarrow D_2 = 5 - 9 + 4$

$\Rightarrow D_2 = 0$

Again, D_3 by replacing 3rd column by B

Here

$$B = \begin{vmatrix} 3 \\ 2 \\ 1 \end{vmatrix}$$

$$\Rightarrow D_3 = \begin{vmatrix} 1 & -1 & 3 \\ 2 & 1 & 2 \\ -1 & -2 & 1 \end{vmatrix}$$

$\Rightarrow D_3 = 1[1 - (-2)(2)] - (-1)[(2)1 - 2(-1)] + 3[2(-2) - 1(-1)]$

$\Rightarrow D_3 = [1 + 4] + [2 + 2] + 3[-4 + 1]$

$\Rightarrow D_3 = 5 + 4 - 9$

$\Rightarrow D_3 = 0$

So, here we can see that

$D = D_1 = D_2 = D_3 = 0$

Thus,

Either the system is consistent with infinitely many solutions or it is inconsistent.

Now, by 1st two equations, written as

$x - y = 3 - z$

$2x + y = 2 + z$

Now by applying Cramer's rule to solve them,

New D and D_1, D_2

$$\Rightarrow D = \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix}$$

$\Rightarrow D = 1 + 2$

$\Rightarrow D = 3$

Again, D_1 by replacing 1st column with

$$B = \begin{vmatrix} 3 - z \\ 2 + z \end{vmatrix}$$

$$\Rightarrow D_1 = \begin{vmatrix} 3 - z & -1 \\ 2 + z & 1 \end{vmatrix}$$

$\Rightarrow D_1 = 3 - z - (-1)(2 + z)$

$\Rightarrow D_1 = 5$

Again, D_2 by replacing 2nd column with

$$B = \begin{vmatrix} 3 - z \\ 2 + z \end{vmatrix}$$

$\Rightarrow D_2 = \begin{vmatrix} 1 & 3-z \\ 2 & 2+z \end{vmatrix}$

$\Rightarrow D_2 = 2 + z - 2(3 - z)$

$\Rightarrow D_2 = -4 + 3z$

Hence, using Cramer's rule

$\Rightarrow x = \frac{D_1}{D}$

$\Rightarrow x = \frac{5}{3}$

again,

$\Rightarrow y = \frac{D_2}{D}$

$\Rightarrow y = \frac{-4+3z}{3}$

Let, z = k

Then $y = \frac{-4+3k}{3}$

And z = k

By changing value of k you may get infinite solutions

27. Question

Show that each of the following systems of linear equations has infinite number of solutions and solve:

x + 2y = 5

3x + 6y = 15

Answer

Given: - Two equation x + 2y = 5 and 3x + 6y = 15

Tip: - We know that

For a system of 2 simultaneous linear equation with 2 unknowns

(iv) If D ≠ 0, then the given system of equations is consistent and has a unique solution given by

$x = \frac{D_1}{D}, y = \frac{D_2}{D}$

(v) If D = 0 and $D_1 = D_2 = 0$, then the system is consistent and has infinitely many solution.

(vi) If D = 0 and one of D_1 and D_2 is non – zero, then the system is inconsistent.

Now,

We have,

x + 2y = 5

3x + 6y = 15

Lets find D

$\Rightarrow D = \begin{vmatrix} 1 & 2 \\ 3 & 6 \end{vmatrix}$

$\Rightarrow D = -6 - 6$

$\Rightarrow D = 0$

Again, D_1 by replacing 1^{st} column by B

Here

$$B = \begin{vmatrix} 5 \\ 15 \end{vmatrix}$$

$$\Rightarrow D_1 = \begin{vmatrix} 5 & 2 \\ 15 & 6 \end{vmatrix}$$

$\Rightarrow D_1 = 30 - 30$

$\Rightarrow D_1 = 0$

And, D_2 by replacing 2^{nd} column by B

Here

$$B = \begin{vmatrix} 5 \\ 15 \end{vmatrix}$$

$$\Rightarrow D_2 = \begin{vmatrix} 1 & 5 \\ 3 & 15 \end{vmatrix}$$

$\Rightarrow D_2 = 15 - 15$

$\Rightarrow D_2 = 0$

So, here we can see that

$D = D_1 = D_2 = 0$

Thus,

The system is consistent with infinitely many solutions.

Let

$y = k$

then,

$\Rightarrow x + 2y = 5$

$\Rightarrow x = 5 - 2k$

By changing value of k you may get infinite solutions

28. Question

Show that each of the following systems of linear equations has infinite number of solutions and solve:

$x + y - z = 0$

$x - 2y + z = 0$

$3x + 6y - 5z = 0$

Answer

Given: - Three equation

$x + y - z = 0$

$x - 2y + z = 0$

$3x + 6y - 5z = 0$

Tip: - We know that

For a system of 3 simultaneous linear equation with 3 unknowns

(i) If $D \neq 0$, then the given system of equations is consistent and has a unique solution given by

$$x = \frac{D_1}{D}, y = \frac{D_2}{D} \text{ and } z = \frac{D_3}{D}$$

(ii) If $D = 0$ and $D_1 = D_2 = D_3 = 0$, then the given system of equation may or may not be consistent. However if consistent, then it has infinitely many solution.

(iii) If $D = 0$ and at least one of the determinants D_1, D_2 and D_3 is non – zero, then the system is inconsistent.

Now,

We have,

$x + y - z = 0$

$x - 2y + z = 0$

$3x + 6y - 5z = 0$

Lets find D

$$\Rightarrow D = \begin{vmatrix} 1 & 1 & -1 \\ 1 & -2 & 1 \\ 3 & 6 & -5 \end{vmatrix}$$

Expanding along 1^{st} row

$\Rightarrow D = 1[10 - (6)1] - (1)[(-5)1 - (1)3] + (-1)[6 - (-2)3]$

$\Rightarrow D = 1[4] - 1[-8] - [12]$

$\Rightarrow D = 0$

Again, D_1 by replacing 1^{st} column by B

Here

$$B = \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix}$$

$$\Rightarrow D_1 = \begin{vmatrix} 0 & 1 & -1 \\ 0 & -2 & 1 \\ 0 & 6 & -5 \end{vmatrix}$$

As one column is zero its determinant is zero

$\Rightarrow D_1 = 0$

Also, D_2 by replacing 2^{nd} column by B

Here

$$B = \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix}$$

$$\Rightarrow D_2 = \begin{vmatrix} 1 & 0 & -1 \\ 1 & 0 & 1 \\ 3 & 0 & -5 \end{vmatrix}$$

As one column is zero its determinant is zero

$\Rightarrow D_2 = 0$

Again, D_3 by replacing 3^{rd} column by B

Here

$$B = \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix}$$

$$\Rightarrow D_3 = \begin{vmatrix} 1 & 1 & 0 \\ 1 & -2 & 0 \\ 3 & 6 & 0 \end{vmatrix}$$

As one column is zero its determinant is zero

$\Rightarrow D_3 = 0$

So, here we can see that

$D = D_1 = D_2 = D_3 = 0$

Thus,

Either the system is consistent with infinitely many solutions or it is inconsistent.

Now, by 1^{st} two equations, written as

$x + y = z$

$x - 2y = - z$

Now by applying Cramer's rule to solve them,

New D and D_1, D_2

$$\Rightarrow D = \begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix}$$

$\Rightarrow D = - 2 - 1$

$\Rightarrow D = - 3$

Again, D_1 by replacing 1^{st} column with

$$B = \begin{vmatrix} z \\ -z \end{vmatrix}$$

$$\Rightarrow D_1 = \begin{vmatrix} z & 1 \\ -z & -2 \end{vmatrix}$$

$\Rightarrow D_1 = - 2z - 1(- z)$

$\Rightarrow D_1 = - z$

Again, D_2 by replacing 2^{nd} column with

$$B = \begin{vmatrix} z \\ -z \end{vmatrix}$$

$$\Rightarrow D_2 = \begin{vmatrix} 1 & z \\ 1 & -z \end{vmatrix}$$

$\Rightarrow D_2 = - z - z$

$\Rightarrow D_2 = - 2z$

Hence, using Cramer's rule

$$\Rightarrow x = \frac{D_1}{D}$$

$$\Rightarrow x = \frac{-z}{-3}$$

Let, $z = k$

Then $x = \frac{k}{3}$

again,

$\Rightarrow y = \frac{D_2}{D}$

$\Rightarrow y = \frac{-2z}{-3}$

$\Rightarrow y = \frac{2k}{3}$

And z = k

By changing value of k you may get infinite solutions

29. Question

Show that each of the following systems of linear equations has infinite number of solutions and solve:

2x + y – 2z = 4

x – 2y + z = – 2

5x – 5y + z = – 2

Answer

Given: - Three equation

2x + y – 2z = 4

x – 2y + z = – 2

5x – 5y + z = – 2

Tip: - We know that

For a system of 3 simultaneous linear equation with 3 unknowns

(i) If D ≠ 0, then the given system of equations is consistent and has a unique solution given by

$x = \frac{D_1}{D}, y = \frac{D_2}{D} \text{ and } z = \frac{D_3}{D}$

(ii) If D = 0 and $D_1 = D_2 = D_3 = 0$, then the given system of equation may or may not be consistent. However if consistent, then it has infinitely many solution.

(iii) If D = 0 and at least one of the determinants D_1, D_2 and D_3 is non – zero, then the system is inconsistent.

Now,

We have,

2x + y – 2z = 4

x – 2y + z = – 2

5x – 5y + z = – 2

Lets find D

$$\Rightarrow D = \begin{vmatrix} 2 & 1 & -2 \\ 1 & -2 & 1 \\ 5 & -5 & 1 \end{vmatrix}$$

Expanding along 1st row

⇒ D = 2[– 2 – (– 5)(1)] – (1)[(1)1 – 5(1)] + (– 2)[– 5 – 5(– 2)]

⇒ D = 2[3] – 1[– 4] – 2[5]

$\Rightarrow D = 0$

Again, D_1 by replacing 1^{st} column by B

Here

$$B = \begin{vmatrix} 4 \\ -2 \\ -2 \end{vmatrix}$$

$$\Rightarrow D_1 = \begin{vmatrix} 4 & 1 & -2 \\ -2 & -2 & 1 \\ -2 & -5 & 1 \end{vmatrix}$$

$\Rightarrow D_1 = 4[-2-(-5)(1)] - (1)[(-2)1 - (-2)(1)] + (-2)[(-2)(-5) - (-2)(-2)]$

$\Rightarrow D_1 = 4[-2+5] - [-2+2] - 2[6]$

$\Rightarrow D_1 = 12 + 0 - 12$

$\Rightarrow D_1 = 0$

Also, D_2 by replacing 2^{nd} column by B

Here

$$B = \begin{vmatrix} 4 \\ -2 \\ -2 \end{vmatrix}$$

$$\Rightarrow D_2 = \begin{vmatrix} 2 & 4 & -2 \\ 1 & -2 & 1 \\ 5 & -2 & 1 \end{vmatrix}$$

$\Rightarrow D_2 = 2[-2-(-2)(1)] - (4)[(1)1 - (5)] + (-2)[-2-5(-2)]$

$\Rightarrow D_2 = 2[-2+2] - 4[-4] + (-2)[8]$

$\Rightarrow D_2 = 0 + 16 - 16$

$\Rightarrow D_2 = 0$

Again, D_3 by replacing 3^{rd} column by B

Here

$$B = \begin{vmatrix} 4 \\ -2 \\ -2 \end{vmatrix}$$

$$\Rightarrow D_3 = \begin{vmatrix} 2 & 1 & 4 \\ 1 & -2 & -2 \\ 5 & -5 & -2 \end{vmatrix}$$

$\Rightarrow D_3 = 2[4-(-2)(-5)] - (1)[(-2)1 - 5(-2)] + 4[1(-5) - 5(-2)]$

$\Rightarrow D_3 = 2[-6] - [8] + 4[-5+10]$

$\Rightarrow D_3 = -12 - 8 + 20$

$\Rightarrow D_3 = 0$

So, here we can see that

$D = D_1 = D_2 = D_3 = 0$

Thus,

Either the system is consistent with infinitely many solutions or it is inconsistent.

Now, by 1^{st} two equations, written as

x – 2y = – 2 – z

5x – 5y = – 2 – z

Now by applying Cramer's rule to solve them,

New D and D_1, D_2

$\Rightarrow D = \begin{vmatrix} 1 & -2 \\ 5 & -5 \end{vmatrix}$

⇒ D = – 5 + 10

⇒ D = 5

Again, D_1 by replacing 1^{st} column with

$B = \begin{vmatrix} -2-z \\ -2-z \end{vmatrix}$

$\Rightarrow D_1 = \begin{vmatrix} -2-z & -2 \\ -2-z & -5 \end{vmatrix}$

⇒ D_1 = 10 + 5z – (– 2)(– 2 – z)

⇒ D_1 = 6 + 3z

Again, D_2 by replacing 2^{nd} column with

$B = \begin{vmatrix} -2-z \\ -2-z \end{vmatrix}$

$\Rightarrow D_2 = \begin{vmatrix} 1 & -2-z \\ 5 & -2-z \end{vmatrix}$

⇒ D_2 = – 2 – z – 5 (– 2 – z)

⇒ D_2 = 8 + 4z

Hence, using Cramer's rule

$\Rightarrow x = \frac{D_1}{D}$

$\Rightarrow x = \frac{6+3z}{5}$

again,

$\Rightarrow y = \frac{D_2}{D}$

$\Rightarrow y = \frac{8+4z}{5}$

Let, z = k

Then

$$x = \frac{6 + 3k}{5}$$

$$y = \frac{8 + 4k}{5}$$

And z = k

By changing value of k you may get infinite solutions

30. Question

Show that each of the following systems of linear equations has infinite number of solutions and solve:

x – y + 3z = 6

x + 3y – 3z = – 4

5x + 3y + 3z = 10

Answer

Given: - Three equation

x – y + 3z = 6

x + 3y – 3z = – 4

5x + 3y + 3z = 10

Tip: - We know that

For a system of 3 simultaneous linear equation with 3 unknowns

(iv) If D ≠ 0, then the given system of equations is consistent and has a unique solution given by

$$x = \frac{D_1}{D}, y = \frac{D_2}{D} \text{ and } z = \frac{D_3}{D}$$

(v) If D = 0 and $D_1 = D_2 = D_3 = 0$, then the given system of equation may or may not be consistent. However if consistent, then it has infinitely many solution.

(vi) If D = 0 and at least one of the determinants D_1, D_2 and D_3 is non – zero, then the system is inconsistent.

Now,

We have,

x – y + 3z = 6

x + 3y – 3z = – 4

5x + 3y + 3z = 10

Lets find D

$$\Rightarrow D = \begin{vmatrix} 1 & -1 & 3 \\ 1 & 3 & -3 \\ 5 & 3 & 3 \end{vmatrix}$$

Expanding along 1^{st} row

⇒ D = 1[9 – (– 3)(3)] – (– 1)[(3)1 – 5(– 3)] + 3[3 – 5(3)]

⇒ D = 1[18] + 1[18] + 3[12]

⇒ D = 0

Again, D_1 by replacing 1^{st} column by B

Here

$$B = \begin{vmatrix} 6 \\ -4 \\ 10 \end{vmatrix}$$

$$\Rightarrow D_1 = \begin{vmatrix} 6 & -1 & 3 \\ -4 & 3 & -3 \\ 10 & 3 & 3 \end{vmatrix}$$

⇒ D_1 = 6[9 – (– 3)(3)] – (– 1)[(– 4)3 – 10(– 3)] + 3[– 12 – 30]

$\Rightarrow D_1 = 6[9 + 9] + [-12 + 30] + 3[-42]$

$\Rightarrow D_1 = 6[18] + 18 - 3[42]$

$\Rightarrow D_1 = 0$

Also, D_2 by replacing 2^{nd} column by B

Here

$$B = \begin{vmatrix} 6 \\ -4 \\ 10 \end{vmatrix}$$

$$\Rightarrow D_2 = \begin{vmatrix} 1 & 6 & 3 \\ 1 & -4 & -3 \\ 5 & 10 & 3 \end{vmatrix}$$

$\Rightarrow D_2 = 1[-12 - (-3)10] - 6[3 - 5(-3)] + 3[10 - 5(-4)]$

$\Rightarrow D_2 = [-12 + 30] - 6[3 + 15] + 3[10 + 20]$

$\Rightarrow D_2 = 18 - 6[18] + 3[30]$

$\Rightarrow D_2 = 0$

Again, D_3 by replacing 3^{rd} column by B

Here

$$B = \begin{vmatrix} 6 \\ -4 \\ 10 \end{vmatrix}$$

$$\Rightarrow D_3 = \begin{vmatrix} 1 & -1 & 6 \\ 1 & 3 & -4 \\ 5 & 3 & 10 \end{vmatrix}$$

$\Rightarrow D_3 = 1[30 - (-4)(3)] - (-1)[(10 - 5(-4)] + 6[3 - 15]$

$\Rightarrow D_3 = 1[30 + 12] + 1[10 + 20] + 6[-12]$

$\Rightarrow D_3 = 42 + 30 - 72$

$\Rightarrow D_3 = 0$

So, here we can see that

$D = D_1 = D_2 = D_3 = 0$

Thus,

Either the system is consistent with infinitely many solutions or it is inconsistent.

Now, by 1^{st} two equations, written as

$x - y = 6 - 3z$

$x + 3y = -4 + 3z$

Now by applying Cramer's rule to solve them,

New D and D_1, D_2

$$\Rightarrow D = \begin{vmatrix} 1 & -1 \\ 1 & 3 \end{vmatrix}$$

$\Rightarrow D = 3 + 1$

$\Rightarrow D = 4$

Again, D_1 by replacing 1^{st} column with

$$B = \begin{vmatrix} 6 - 3z \\ -4 + 3z \end{vmatrix}$$

$$\Rightarrow D_1 = \begin{vmatrix} 6 - 3z & -1 \\ -4 + 3z & 3 \end{vmatrix}$$

$\Rightarrow D_1 = 18 - 9z - (-1)(-4 + 3z)$

$\Rightarrow D_1 = 14 - 5z$

Again, D_2 by replacing 2^{nd} column with

$$B = \begin{vmatrix} 6 - 3z \\ -4 + 3z \end{vmatrix}$$

$$\Rightarrow D_2 = \begin{vmatrix} 1 & 6 - 3z \\ 1 & -4 + 3z \end{vmatrix}$$

$\Rightarrow D_2 = -4 + 3z - (6 - 3z)$

$\Rightarrow D_2 = -10 + 6z$

Hence, using Cramer's rule

$$\Rightarrow x = \frac{D_1}{D}$$

$$\Rightarrow x = \frac{14-6z}{4}$$

$$\Rightarrow x = \frac{7-3z}{2}$$

again,

$$\Rightarrow y = \frac{D_2}{D}$$

$$\Rightarrow y = \frac{-10+6z}{4}$$

$$\Rightarrow y = \frac{-5+3z}{2}$$

Let, z = k

Then

$$x = \frac{7 - 3k}{2}$$

$$y = \frac{-5 + 3k}{2}$$

And z = k

By changing value of k you may get infinite solutions

31. Question

A salesman has the following record of sales during three months for three items A,B and C which have different rates of commission.

Month	Sales of Units			Total commission drawn (in ₹)
	A	B	C	
Jan	90	100	20	800
Feb	130	50	40	900
March	60	100	30	850

Find out the rates of commission on items A,B and C by using determinant method.

Answer

Given: - Record of sales during three months

Let, rates of commissions on items A,B and C be x, y and z respectively.

Now, we can arrange this model in linear equation system

Thus, we have

$90x + 100y + 20z = 800$

$130x + 50y + 40z = 900$

$60x + 100y + 30z = 850$

Here

$$\Rightarrow D = \begin{vmatrix} 90 & 100 & 20 \\ 130 & 50 & 40 \\ 60 & 100 & 30 \end{vmatrix}$$

Applying, $r_1 \rightarrow r_1 - 2r_2, r_3 \rightarrow r_3 - 2r_2$

$$\Rightarrow D = \begin{vmatrix} -170 & 0 & -60 \\ 130 & 50 & 40 \\ -200 & 0 & -50 \end{vmatrix}$$

Solving determinant, expanding along 2^{nd} column

$\Rightarrow D = 50[(-50)(-170) - (-200)(-60)]$

$\Rightarrow D = 50[8500 - 12000]$

$\Rightarrow D = -175000$

Again, Solve D_1 formed by replacing 1^{st} column by B matrices

Here

$$B = \begin{vmatrix} 800 \\ 900 \\ 850 \end{vmatrix}$$

$$\Rightarrow D_1 = \begin{vmatrix} 800 & 100 & 20 \\ 900 & 50 & 40 \\ 850 & 100 & 30 \end{vmatrix}$$

Applying, $r_1 \rightarrow r_1 - 2r_2, r_3 \rightarrow r_3 - 2r_2$

$$\Rightarrow D_1 = \begin{vmatrix} -1000 & 0 & -60 \\ 900 & 50 & 40 \\ -950 & 0 & -500 \end{vmatrix}$$

Solving determinant, expanding along 2^{nd} column

$\Rightarrow D_1 = 50[(-1000)(-500) - (-950)(-60)]$

$\Rightarrow D_1 = 50[50000 - 57000]$

$\Rightarrow D_1 = -350000$

Again, Solve D_2 formed by replacing 2^{nd} column by B matrices

Here

$$B = \begin{vmatrix} 800 \\ 900 \\ 850 \end{vmatrix}$$

$$\Rightarrow D_2 = \begin{vmatrix} 90 & 800 & 20 \\ 130 & 900 & 40 \\ 60 & 850 & 30 \end{vmatrix}$$

Applying, $r_2 \to r_2 - 2r_1$

$$\Rightarrow D_2 = \begin{vmatrix} 90 & 800 & 20 \\ -50 & -700 & 0 \\ -75 & -350 & 0 \end{vmatrix}$$

Solving determinant, expanding along 1st Row

$\Rightarrow D_2 = 20[17500 - 52500]$

$\Rightarrow D_2 = -700000$

And, Solve D_3 formed by replacing 3rd column by B matrices

Here

$$B = \begin{vmatrix} 800 \\ 900 \\ 850 \end{vmatrix}$$

$$\Rightarrow D_3 = \begin{vmatrix} 90 & 100 & 800 \\ 130 & 50 & 900 \\ 60 & 100 & 850 \end{vmatrix}$$

Applying, $r_1 \to r_1 - 2r_2, r_3 \to r_3 - 2r_2$

$$\Rightarrow D_3 = \begin{vmatrix} -170 & 0 & -1000 \\ 130 & 50 & 900 \\ -200 & 0 & -950 \end{vmatrix}$$

Solving determinant, expanding along 1st Row

$\Rightarrow D_3 = 50[161500 - 200000]$

$\Rightarrow D_3 = -1925000$

Thus by Cramer's Rule, we have

$$\Rightarrow x = \frac{D_1}{D}$$

$$\Rightarrow x = \frac{-350000}{-175000}$$

$\Rightarrow x = 2$

again,

$$\Rightarrow y = \frac{D_2}{D}$$

$$\Rightarrow y = \frac{-700000}{-175000}$$

$\Rightarrow y = 4$

and,

$$\Rightarrow z = \frac{D_3}{D}$$

$$\Rightarrow z = \frac{-1925000}{-175000}$$

z = 11

Thus rates of commission of items A, B and C are 2%, 4% and 11% respectively.

32. Question

An automobile company uses three types of steel S_1, S_2 and S_3 for producing three types of cars C_1, C_2 and C_3. Steel requirements (in tons) for each type of cars are given below:

cars | Steel			
	C_1	C_2	C_3
S_1	2	3	4
S_2	1	1	2
S_3	3	2	1

Using Cramer's rule, find the number of cars of each type which can be produced using 29, 13 and 16 tonnes of steel of three types respectively.

Answer

Given: - Steel requirement for each car is given

Let, Number of cars produced by steel type C_1, C_2 and C_3 be x, y and z respectively.

Now, we can arrange this model in linear equation system

Thus, we have

2x + 3y + 4z = 29

x + y + 2z = 13

3x + 2y + z = 16

Here

$$\Rightarrow D = \begin{vmatrix} 2 & 3 & 4 \\ 1 & 1 & 2 \\ 3 & 2 & 1 \end{vmatrix}$$

Applying, $r_1 \to r_1 - 4r_3, r_2 \to r_2 - 2r_3$

$$\Rightarrow D = \begin{vmatrix} -10 & -5 & 0 \\ -5 & -3 & 0 \\ 3 & 2 & 1 \end{vmatrix}$$

Solving determinant, expanding along 3^{rd} column

⇒ D = 1[30 – 25]

⇒ D = 5

⇒ D = 5

Again, Solve D_1 formed by replacing 1^{st} column by B matrices

Here

$$B = \begin{vmatrix} 29 \\ 13 \\ 16 \end{vmatrix}$$

$$\Rightarrow D_1 = \begin{vmatrix} 29 & 3 & 4 \\ 13 & 1 & 2 \\ 16 & 2 & 1 \end{vmatrix}$$

Applying, $r_1 \to r_1 - 4r_3, r_2 \to r_2 - 2r_3$

$\Rightarrow D_1 = \begin{vmatrix} -35 & -5 & 0 \\ -19 & -3 & 0 \\ 16 & 2 & 1 \end{vmatrix}$

Solving determinant, expanding along 3rd column

⇒ D_1 = 1[(– 35)(– 3) – (– 5)(– 19)]

⇒ D_1 = 1[105 – 95]

⇒ D_1 = 10

Again, Solve D_2 formed by replacing 2nd column by B matrices

Here

$B = \begin{vmatrix} 29 \\ 13 \\ 16 \end{vmatrix}$

$\Rightarrow \ D_2 = \begin{vmatrix} 2 & 29 & 4 \\ 1 & 13 & 2 \\ 3 & 16 & 1 \end{vmatrix}$

Applying, $r_1 \rightarrow r_1 - 4r_3, r_2 \rightarrow \ r_2 - 2r_3$

$\Rightarrow D_2 = \begin{vmatrix} -10 & -35 & 0 \\ -5 & -19 & 0 \\ 3 & 16 & 1 \end{vmatrix}$

Solving determinant, expanding along 3rd column

⇒ D_2 = 1[190 – 175]

⇒ D_2 = 15

And, Solve D_3 formed by replacing 3rd column by B matrices

Here

$B = \begin{vmatrix} 29 \\ 13 \\ 16 \end{vmatrix}$

$\Rightarrow D_3 = \begin{vmatrix} 2 & 3 & 29 \\ 1 & 1 & 13 \\ 3 & 2 & 16 \end{vmatrix}$

Applying, $r_1 \rightarrow r_1 - 2r_2, r_3 \rightarrow \ r_3 - 3r_2$

$\Rightarrow D_3 = \begin{vmatrix} 0 & 1 & 3 \\ 1 & 1 & 13 \\ 0 & -1 & -23 \end{vmatrix}$

Solving determinant, expanding along 1st column

⇒ D_3 = – 1[– 23 – (– 1)3]

⇒ D_3 = 20

Thus by Cramer’s Rule, we have

$\Rightarrow x = \frac{D_1}{D}$

$\Rightarrow x = \frac{10}{5}$

⇒ x = 2

again,

$\Rightarrow y = \frac{D_2}{D}$

$\Rightarrow y = \frac{15}{5}$

$\Rightarrow$ y = 3

and,

$\Rightarrow z = \frac{D_3}{D}$

$\Rightarrow z = \frac{20}{5}$

$\Rightarrow$ z = 4

Thus Number of cars produced by type C_1, C_2 and C_3 are 2, 3 and 4 respectively.

Exercise 6.5

1. Question

Solve each of the following systems of homogeneous linear equations:

x + y – 2z = 0

2x + y – 3z = 0

5x + 4y – 9z = 0

Answer

Given Equations:

x + y – 2z = 0

2x + y – 3z = 0

5x + 4y – 9z = 0

Any system of equation can be written in matrix form as AX = B

Now finding the Determinant of these set of equations,

$$D = \begin{vmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{vmatrix}$$

$$|A| = 1\begin{vmatrix} 1 & -3 \\ 4 & -9 \end{vmatrix} - 1\begin{vmatrix} 2 & -3 \\ 5 & -9 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ 5 & 4 \end{vmatrix}$$

= 1(1×(– 9) – 4×(– 3)) – 1(2×(– 9) – 5×(– 3)) – 2(4×2 – 5×1)

= 1(– 9 + 12) – 1(– 18 + 15) – 2(8 – 5)

= 1×3 –1 × (– 3) – 2×3

= 3 + 3 – 6

= 0

Since D = 0, so the system of equation has infinite solution.

Now let z = k

$\Rightarrow$ x + y = 2k

And 2x + y = 3k

Now using the cramer’s rule

$$x = \frac{D_1}{D}$$

$$x = \frac{\begin{vmatrix} 2k & 1 \\ 3k & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}}$$

$$x = \frac{-k}{-1}$$

x = k

similarly,

$$y = \frac{D_2}{D}$$

$$y = \frac{\begin{vmatrix} 1 & 2k \\ 2 & 3k \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}}$$

$$y = \frac{-k}{-1}$$

y = k

Hence, x = y = z = k.

2. Question

Solve each of the following systems of homogeneous linear equations:

2x + 3y + 4z = 0

X + y + z = 0

2x + 5y – 2z = 0

Answer

Given Equations:

2x + 3y + 4z = 0

X + y + z = 0

2x + 5y – 2z = 0

Any system of equation can be written in matrix form as AX = B

Now finding the Determinant of these set of equations,

$$D = \begin{vmatrix} 2 & 3 & 4 \\ 1 & 1 & 1 \\ 2 & 5 & -2 \end{vmatrix}$$

$$|A| = 2\begin{vmatrix} 1 & 1 \\ 5 & -2 \end{vmatrix} - 3\begin{vmatrix} 1 & 1 \\ 2 & -2 \end{vmatrix} + 4\begin{vmatrix} 1 & 1 \\ 2 & 5 \end{vmatrix}$$

= 2(1×(– 2) – 1×5) – 3(1×(– 2) – 2×1) + 4(1×5 – 2×1)

= 2(– 2 – 5) – 3(– 2 – 2) + 4(5 – 2)

= 1×(– 7) – 3 × (– 4) + 4×3

= – 7 + 12 + 12

= 17

Since D ≠ 0, so the system of equation has infinite solution.

Therefore the system of equation has only solution as x = y = z = 0.

3. Question

Solve each of the following systems of homogeneous linear equations:

3x + y + z = 0

x – 4y3z = 0

2x + 5y – 2z = 0

Answer

Given Equations:

3x + y + z = 0

x – 4y + 3z = 0

2x + 5y – 2z = 0

Any system of equation can be written in matrix form as AX = B

Now finding the Determinant of these set of equations,

$$D = \begin{vmatrix} 3 & 1 & 1 \\ 1 & -4 & 3 \\ 2 & 5 & -2 \end{vmatrix}$$

$$|D| = 3\begin{vmatrix} -4 & 3 \\ 5 & -2 \end{vmatrix} - 1\begin{vmatrix} 1 & 3 \\ 2 & -2 \end{vmatrix} + 1\begin{vmatrix} 1 & -4 \\ 2 & 5 \end{vmatrix}$$

= 3(– 4×(– 2) – 3×5) – 1(1×(– 2) – 3×2) + 1(1×5 – 2×(– 4))

= 3(8 – 15) – 1(– 2 – 6) + 1(5 + 8)

= 3×(– 7) –1 × (– 8) + 1×13

= – 21 + 8 + 13

= 0

Since D = 0, so the system of equation has infinite solution.

Now let z = k

⇒ 3x + y = – k

And x – 4y = – 3k

Now using the cramer’s rule

$$x = \frac{D_1}{D}$$

$$x = \frac{\begin{vmatrix} -k & 1 \\ -3k & -4 \end{vmatrix}}{\begin{vmatrix} 3 & 1 \\ 1 & -4 \end{vmatrix}}$$

$$x = \frac{7k}{-13}$$

similarly,

$$y = \frac{D_2}{D}$$

$$y = \frac{\begin{vmatrix} 3 & -k \\ 1 & -3k \end{vmatrix}}{\begin{vmatrix} 3 & 1 \\ 1 & -4 \end{vmatrix}}$$

$$y = \frac{-8k}{-13}$$

Hence $x = \frac{7k}{-13}$; $y = \frac{8k}{13}$ and $z = k$

4. Question

Find the real values of λ for which the followings system of linear equations has non – trivial solutions. Also, find the non – trivial solutions

$2\lambda x - 2y + 3z = 0$

$X + \lambda y + 2z = 0$

$2x + \lambda z = 0$

Answer

Given Equations:

$2\lambda x - 2y + 3z = 0$

$x + \lambda y + 2z = 0$

$2x + \lambda z = 0$

For trivial solution D = 0

$$D = \begin{vmatrix} 2\lambda & -2 & 3 \\ 1 & \lambda & 2 \\ 2 & 0 & \lambda \end{vmatrix}$$

$$|D| = 2\lambda \begin{vmatrix} \lambda & 2 \\ 0 & \lambda \end{vmatrix} - 2 \begin{vmatrix} 1 & 2 \\ 2 & \lambda \end{vmatrix} + 3 \begin{vmatrix} 1 & \lambda \\ 2 & 0 \end{vmatrix}$$

$= 2\lambda (\lambda\times\lambda - 0\times2) + 2(1\times\lambda - 2\times2) + 3(1\times0 - 2\times\lambda)$

$= 2\lambda (\lambda^2 - 0) + 2(\lambda - 4) + 3(0 - 2\lambda)$

$= 2\lambda^3 + 2\lambda - 8 - 6\lambda$

$= 2\lambda^3 + 4\lambda - 8$

Now D = 0

$2\lambda^3 - 4\lambda - 8 = 0$

$2\lambda^3 - 4\lambda = 8$

$\lambda(\lambda^2 - 2) = 4$

Hence λ = 2

Now let z = k

$\Rightarrow 4x - 2y = -3k$

And $x + 2y = -2k$

Now using the cramer's rule

$$x = \frac{D_1}{D}$$

$$x = \frac{\begin{vmatrix} -3k & -2 \\ -2k & 2 \end{vmatrix}}{\begin{vmatrix} 4 & -2 \\ 1 & 2 \end{vmatrix}}$$

$$x = \frac{-10k}{10}$$

x = – k

similarly,

$$y = \frac{D_2}{D}$$

$$y = \frac{\begin{vmatrix} 4 & -3k \\ 1 & -2k \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}}$$

$$y = \frac{-5k}{10}$$

$$y = -\frac{k}{2}$$

$$\text{Hence } x = -k\,; y = -\frac{k}{2} \text{ and } z = k$$

5. Question

If a,b,c are non – zero real numbers and if the system of equations

(a – 1) x = y + z

(b – 1) y = z + x

(c – 1) z = x + y

Has a non – trivial solution, then prove that ab + bc + ca = abc.

Answer

Given Equations:

(a – 1) x = y + z

(b – 1) y = z + x

(c – 1) z = x + y

Rearranging these equations

(a – 1)x – y – z = 0

– x + (b – 1)y – z = 0

– x – y + (c – 1)z = 0

For trivial solution D = 0

$$D = \begin{vmatrix} (a-1) & -1 & -1 \\ -1 & (b-1) & -1 \\ -1 & -1 & (c-1) \end{vmatrix}$$

$$|D| = (a-1)\begin{vmatrix} (b-1) & -1 \\ -1 & (c-1) \end{vmatrix} + 1\begin{vmatrix} -1 & -1 \\ -1 & (c-1) \end{vmatrix} - 1\begin{vmatrix} -1 & (b-1) \\ -1 & -1 \end{vmatrix}$$

= (a – 1)((b – 1)(c – 1) – (– 1)×(– 1)) + 1(– 1(c – 1) – (– 1)×(– 1)) – 1((– 1)×(– 1) + (b – 1))

$= (a - 1)(bc - b - c + 1 - 1) + (1 - c - 1) - 1(1 + b - 1))$

$= (a - 1)(bc - b - c) - c - b$

$= abc - ab - ac - bc + b + c - b - c$

$= abc - ab - ac - bc$

Now D = 0

$\Rightarrow abc - ab - ac - bc = 0$

$\Rightarrow abc = ab + bc + ac$

Hence proved.

MCQ

1. Question

Mark the correct alternative in the following:

If A and B are square matrices or order 2, then det (A + B) = 0 is possible only when

A. det (A) = 0 or det (B) = 0

B. det (A) + det (B) = 0

C. det (A) = 0 and det (B) = 0

D. A + B = 0

Answer

We are given that,

Matrices A and B are square matrices.

Order of matrix A = 2

Order of matrix B = 2

Det (A + B) = 0

We need to find the condition at which det (A + B) = 0.

Let,

Matrix $A = [a_{ij}]$

Matrix $B = [b_{ij}]$

Since their orders are same, we can express matrices A and B as

$A + B = [a_{ij} + b_{ij}]$

$\Rightarrow |A + B| = |a_{ij} + b_{ij}|$ ...(i)

Also, we know that

Det (A + B) = 0

That is, |A + B| = 0

From (i),

$|a_{ij} + b_{ij}| = 0$

If

$\Rightarrow [a_{ij} + b_{ij}] = 0$

Each corresponding element is 0.

$\Rightarrow$ A + B = 0

Thus, det (A + B) = 0 is possible when A + B = 0 .

2. Question

Mark the correct alternative in the following:

Which of the following is not correct?

A. $|A| = |A^T|$, where $A = [a_{ij}]_{3 \times 3}$

B. $|kA| = k^3 |A|$, where $A = [a_{ij}]_{3 \times 3}$

C. If A is a skew-symmetric matrix of odd order, then |A| = 0

D. $\begin{vmatrix} a+b & c+d \\ e+f & g+h \end{vmatrix} = \begin{vmatrix} a & c \\ e & g \end{vmatrix} + \begin{vmatrix} b & d \\ f & h \end{vmatrix}$

Answer

We are given that,

$A = [a_{ij}]_{3\times3}$

That is, order of matrix A = 3

Example:

Let,

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 3 & 2 \\ 3 & 2 & 1 \end{bmatrix}$$

Take determinant of A.

Determinant of 3 × 3 matrices is found as,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}.\det\begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} - a_{12}.\det\begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix} + a_{13}.\det\begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

$$\Rightarrow \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}(a_{22} \times a_{33} - a_{23} \times a_{32}) - a_{12}(a_{21} \times a_{33} - a_{23} \times a_{31}) + a_{13}(a_{21} \times a_{32} - a_{22} \times a_{31})$$

So,

$$\begin{vmatrix} 2 & 1 & 2 \\ 1 & 3 & 2 \\ 3 & 2 & 1 \end{vmatrix} = 2.\det\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} - 1.\det\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} + 2.\det\begin{bmatrix} 1 & 3 \\ 3 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{vmatrix} 2 & 3 & 4 \\ 1 & 3 & 2 \\ 3 & 2 & 1 \end{vmatrix} = 2(3 \times 1 - 2 \times 2) - 1(1 \times 1 - 2 \times 3) + 2(1 \times 2 - 3 \times 3)$$

$$\Rightarrow \begin{vmatrix} 2 & 3 & 4 \\ 1 & 3 & 2 \\ 3 & 2 & 1 \end{vmatrix} = 2(3 - 4) - 1(1 - 6) + 2(2 - 9)$$

$$\Rightarrow \begin{vmatrix} 2 & 3 & 4 \\ 1 & 3 & 2 \\ 3 & 2 & 1 \end{vmatrix} = 2(-1) - (-5) + 2(-7)$$

$$\Rightarrow \begin{vmatrix} 2 & 3 & 4 \\ 1 & 3 & 2 \\ 3 & 2 & 1 \end{vmatrix} = -2 + 5 - 14$$

$$\Rightarrow \begin{vmatrix} 2 & 3 & 4 \\ 1 & 3 & 2 \\ 3 & 2 & 1 \end{vmatrix} = -11$$

⇒ |A| = -11

The transpose of a matrix is a new matrix whose rows are the columns of the original.

So,

$$A^T = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 3 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

Determinant of A^T:

$$\begin{vmatrix} 2 & 1 & 3 \\ 3 & 3 & 2 \\ 4 & 2 & 1 \end{vmatrix} = 2.\det\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} - 1.\det\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} + 3.\det\begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{vmatrix} 2 & 1 & 3 \\ 3 & 3 & 2 \\ 4 & 2 & 1 \end{vmatrix} = 2(3 \times 1 - 2 \times 2) - (1 \times 1 - 2 \times 2) + 3(1 \times 2 - 3 \times 2)$$

$$\Rightarrow \begin{vmatrix} 2 & 1 & 3 \\ 3 & 3 & 2 \\ 4 & 2 & 1 \end{vmatrix} = 2(3 - 4) - (1 - 4) + 3(2 - 6)$$

$$\Rightarrow \begin{vmatrix} 2 & 1 & 3 \\ 3 & 3 & 2 \\ 4 & 2 & 1 \end{vmatrix} = 2(-1) - (-3) + 3(-4)$$

$$\Rightarrow \begin{vmatrix} 2 & 1 & 3 \\ 3 & 3 & 2 \\ 4 & 2 & 1 \end{vmatrix} = -2 + 3 - 12$$

$$\Rightarrow \begin{vmatrix} 2 & 1 & 3 \\ 3 & 3 & 2 \\ 4 & 2 & 1 \end{vmatrix} = -11$$

So, we can conclude that,

$|A| = |A^T|$, where $A = [a_{ij}]_{3\times3}$.

Option (B) is correct.

$|kA| = k^3|A|$, where $A = [a_{ij}]_{3\times3}$

Example:

Let k = 2.

And,

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}$$

Take Left Hand Side of the equation:

LHS = |kA|

$$\Rightarrow \text{LHS} = \left| 2\begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix} \right|$$

Multiply 2 by each term of the matrix.

$$\Rightarrow \text{LHS} = \begin{vmatrix} 2\times2 & 2\times3 & 2\times4 \\ 2\times1 & 2\times2 & 2\times3 \\ 2\times3 & 2\times2 & 2\times1 \end{vmatrix}$$

$$\Rightarrow \text{LHS} = \begin{vmatrix} 4 & 6 & 8 \\ 2 & 4 & 6 \\ 6 & 4 & 2 \end{vmatrix}$$

$$\Rightarrow \text{LHS} = 4.\det\begin{bmatrix} 4 & 6 \\ 4 & 2 \end{bmatrix} - 6.\det\begin{bmatrix} 2 & 6 \\ 6 & 2 \end{bmatrix} + 8.\det\begin{bmatrix} 2 & 4 \\ 6 & 4 \end{bmatrix}$$

⇒ LHS = 4(4 × 2 – 6 × 4) – 6(2 × 2 – 6 × 6) + 8(2 × 4 – 4 × 6)

⇒ LHS = 4(8 – 24) – 6(4 – 36) + 8(8 – 24)

⇒ LHS = 4(-16) – 6(-32) + 8(-16)

⇒ LHS = -64 + 192 – 128

⇒ LHS = 0

Take Right Hand Side of the equation:

RHS = $k^3|A|$

$$\Rightarrow \text{RHS} = 2^3\begin{vmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix}$$

$$\Rightarrow \text{RHS} = 8\begin{vmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix}$$

$$\Rightarrow \text{RHS} = 8\left[2.\det\begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix} - 3.\det\begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} + 4.\det\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}\right]$$

⇒ RHS = 8 [2(2 × 1 – 3 × 2) – 3(1 × 1 – 3 × 3) + 4(1 × 2 – 2 × 3)]

⇒ RHS = 8 [2(2 – 6) – 3(1 – 9) + 4(2 – 6)]

⇒ RHS = 8 [2(-4) – 3(-8) + 4(-4)]

⇒ RHS = 8 [-8 + 24 – 16]

⇒ RHS = 8 × 0

⇒ RHS = 0

Since, LHS = RHS.

We can conclude that,

$|kA| = k^3|A|$, where $A = [a_{ij}]_{3\times3}$

Option (C) is also correct.

If A is a skew-symmetric matrix of odd order, then |A| = 0.

If the transpose of a matrix is equal to the negative of itself, the matrix is said to be skew symmetric. In other words, $A^T = -A$.

Example,

Let a matrix of odd order 3 × 3 be,

$$A = \begin{bmatrix} 0 & -6 & 4 \\ 6 & 0 & 7 \\ -4 & -7 & 0 \end{bmatrix}$$

Take determinant of A.

$$\begin{vmatrix} 0 & -6 & 4 \\ 6 & 0 & 7 \\ -4 & -7 & 0 \end{vmatrix} = 0.\det\begin{bmatrix} 0 & 7 \\ -7 & 0 \end{bmatrix} - (-6).\det\begin{bmatrix} 6 & 7 \\ -4 & 0 \end{bmatrix} + 4.\det\begin{bmatrix} 6 & 0 \\ -4 & -7 \end{bmatrix}$$

$$\Rightarrow \begin{vmatrix} 0 & -6 & 4 \\ 6 & 0 & 7 \\ -4 & -7 & 0 \end{vmatrix} = 0 + 6(6 \times 0 - 7 \times -4) + 4(6 \times (-7) - 0 \times -4)$$

$$\Rightarrow \begin{vmatrix} 0 & -6 & 4 \\ 6 & 0 & 7 \\ -4 & -7 & 0 \end{vmatrix} = 0 + 6(0 + 28) + 4(-42 + 0)$$

$$\Rightarrow \begin{vmatrix} 0 & -6 & 4 \\ 6 & 0 & 7 \\ -4 & -7 & 0 \end{vmatrix} = 0 + 6(28) + 4(-42)$$

$$\Rightarrow \begin{vmatrix} 0 & -6 & 4 \\ 6 & 0 & 7 \\ -4 & -7 & 0 \end{vmatrix} = 168 - 168$$

$$\Rightarrow \begin{vmatrix} 0 & -6 & 4 \\ 6 & 0 & 7 \\ -4 & -7 & 0 \end{vmatrix} = 0$$

Thus, we can conclude that

If A is a skew-symmetric matrix of odd order, then |A| = 0.

Option (D) is incorrect.

Let a = 1, b = 3, c = 3, d = -4, e = -2, f = 5, g = 0 and h = 2.

Take Left Hand Side,

$$\mathrm{LHS} = \begin{vmatrix} a+b & c+d \\ e+f & g+h \end{vmatrix}$$

$$\Rightarrow \mathrm{LHS} = \begin{vmatrix} 1+3 & 3-4 \\ -2+5 & 0+2 \end{vmatrix}$$

$$\Rightarrow \mathrm{LHS} = \begin{vmatrix} 4 & -1 \\ 3 & 2 \end{vmatrix}$$

⇒ LHS = 4 × 2 – (-1) × 3

⇒ LHS = 8 + 3

⇒ LHS = 11

Take Right Hand Side,

$$\mathrm{RHS} = \begin{vmatrix} a & c \\ e & g \end{vmatrix} + \begin{vmatrix} b & d \\ f & h \end{vmatrix}$$

$$\Rightarrow \mathrm{RHS} = \begin{vmatrix} 1 & 3 \\ -2 & 0 \end{vmatrix} + \begin{vmatrix} 3 & -4 \\ 5 & 2 \end{vmatrix}$$

⇒ RHS = (1 × 0 – 3 × (-2)) + (3 × 2 – (-4) × 5)

⇒ RHS = (0 + 6) + (6 + 20)

⇒ RHS = 6 + 26

⇒ RHS = 32

Since, LHS ≠ RHS. Then, we can conclude that,

$$\begin{vmatrix} a+b & c+d \\ e+f & g+h \end{vmatrix} \neq \begin{vmatrix} a & c \\ e & g \end{vmatrix} + \begin{vmatrix} b & d \\ f & h \end{vmatrix}$$

3. Question

Mark the correct alternative in the following:

If $A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ and C_{ij} is cofactor of a_{ij} in A, then value of |A| is given by

A. $a_{11}C_{31} + a_{12}C_{32} + a_{13}C_{33}$

B. $a_{11}C_{11} + a_{12}C_{21} + a_{13}C_{31}$

C. $a_{21}C_{11} + a_{22}C_{12} + a_{23}C_{13}$

D. $a_{11}C_{11} + a_{21}C_{21} + a_{31}C_{31}$

Answer

Let us understand what cofactor of an element is.

A cofactor is the number you get when you remove the column and row of a designated element in a matrix, which is just a numerical grid in the form of a rectangle or a square. The cofactor is always preceded by a positive (+) or negative (-) sign, depending whether the element is in a + or - position. It is

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

Let us recall how to find the cofactor of any element:

If we are given with,

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Cofactor of any element say a_{11} is found by eliminating first row and first column.

$$\text{Cofactor of } a_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$$

⇒ Cofactor of $a_{11} = a_{22} \times a_{33} - a_{23} \times a_{32}$

The sign of cofactor of a_{11} is (+).

And, cofactor of any element, say a_{12} is found by eliminating first row and second column.

$$\text{Cofactor of } a_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

⇒ Cofactor of $a_{12} = a_{21} \times a_{33} - a_{23} \times a_{31}$

The sign of cofactor of a_{12} is (-).

We are given that,

$$A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

And C_{ij} is the cofactor of a_{ij} in A.

Determinant of 3 × 3 matrix is given as,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}.\det\begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} - a_{12}.\det\begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix} + a_{13}.\det\begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

Or,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}.\det\begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} + a_{21}.\det\begin{bmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{bmatrix} + a_{31}.\det\begin{bmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{bmatrix}$$

Or using the definition of cofactors,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}C_{11} + a_{21}C_{21} + a_{31}C_{31}$$

Thus, proved.

4. Question

Mark the correct alternative in the following:

Which of the following is not correct in a given determinant of A, where $A = [a_{ij}]_{3 \times 3}$.

A. Order of minor is less than order of the det (A)

B. Minor of an element can never be equal to cofactor of the same element

C. Value of a determinant is obtained by multiplying elements of a row or column by corresponding cofactors

D. Order of minors and cofactors of elements of A is same

Answer

For option (A),

A minor is the determinant of the square matrix formed by deleting one row and one column from some larger square matrix.

So, the order of minor is always less than the order of determinant.

Thus, option (A) is correct.

For option (B),

A cofactor is the number you get when you remove the column and row of a designated element in a matrix, which is just a numerical grid in the form of a rectangle or a square.

A minor is the determinant of the square matrix formed by deleting one row and one column from some larger square matrix.

Since, the definition of cofactor and minor is same, then we can conclude that

Minor of an element is always equal to cofactor of the same element.

Thus, option (B) is incorrect.

For option (C),

Determinant of 3 × 3 matrix is given as,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}.\det\begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} - a_{12}.\det\begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix} + a_{13}.\det\begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

Or,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}.\det\begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} + a_{21}.\det\begin{bmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{bmatrix} + a_{31}.\det\begin{bmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{bmatrix}$$

Or using the definition of cofactors,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}C_{11} + a_{21}C_{21} + a_{31}C_{31}$$

Thus, option (C) is correct.

For option (D),

A cofactor is the number you get when you remove the column and row of a designated element in a matrix, which is just a numerical grid in the form of a rectangle or a square.

A minor is the determinant of the square matrix formed by deleting one row and one column from some larger square matrix.

Since, the definition of cofactor and minor is same, then we can say that,

Minor of an element is always equal to cofactor of the same element.

⇒ The order of the minor and cofactor of A is same. (where A is some matrix)

Thus, option (D) is correct.

5. Question

Mark the correct alternative in the following:

Let $\begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix} = ax^4 + bx^3 + cx^2 + dx + e$. Then, the value of 5a + 4b + 3c + 2d + e is equal to

A. 0

B. -16

C. 16

D. none of these

Answer

We are given that,

$$\begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix} = ax^4 + bx^3 + cx^2 + dx + e$$

We need to find the value of 5a + 4b + 3c + 2d + e.

Determinant of 3 × 3 matrix is given as,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}.\det\begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} - a_{12}.\det\begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix} + a_{13}.\det\begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

$$\Rightarrow \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}(a_{22} \times a_{33} - a_{23} \times a_{32}) - a_{12}(a_{21} \times a_{33} - a_{23} \times a_{31}) + a_{13}(a_{21} \times a_{32} - a_{22} \times a_{31})$$

So,

$$\begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix} = x.\det\begin{bmatrix} x & 6 \\ x & 6 \end{bmatrix} - 2.\det\begin{bmatrix} x^2 & 6 \\ x & 6 \end{bmatrix} + x.\det\begin{bmatrix} x^2 & x \\ x & x \end{bmatrix}$$

$$\Rightarrow \begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix} = x(x \times 6 - 6 \times x) - 2(x^2 \times 6 - 6 \times x) + x(x^2 \times x - x \times x)$$

$$\Rightarrow \begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix} = x(6x - 6x) - 2(6x^2 - 6x) + x(x^3 - x^2)$$

$$\Rightarrow \begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix} = x(0) - 12x^2 + 12x + x^4 - x^3$$

$$\Rightarrow \begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix} = x^4 - x^3 - 12x^2 + 12x$$

Since,

$$\begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix} = ax^4 + bx^3 + cx^2 + dx + e$$

$\Rightarrow x^4 - x^3 - 12x^2 + 12x = ax^4 + bx^3 + cx^2 + dx + e$

Comparing the left hand side and right hand side of the equation, we get

a = 1

b = -1

c = -12

d = 12

e = 0

Putting these values in 5a + 4b + 3c + 2d + e, we get

5a + 4b + 3c + 2d + e = 5(1) + 4(-1) + 3(-12) + 2(12) + 0

⇒ 5a + 4b + 3c + 2d + e = 5 – 4 – 36 + 24

⇒ 5a + 4b + 3c + 2d + e = 25 – 36

⇒ 5a + 4b + 3c + 2d + e = -11

Thus, the values of 5a + 4b + 3c + 2d + e is -11.

6. Question

Mark the correct alternative in the following:

The value of the determinant $\begin{vmatrix} a^2 & a & 1 \\ \cos nx & \cos(n+1)x & \cos(n+2)x \\ \sin nx & \sin(n+1)x & \sin(n+2)x \end{vmatrix}$ is independent of

A. n

B. a

C. x

D. none of these

Answer

Let us solve the determinant.

$$\begin{vmatrix} a^2 & a & 1 \\ \cos nx & \cos(n+1)x & \cos(n+2)x \\ \sin nx & \sin(n+1)x & \sin(n+2)x \end{vmatrix}$$

We know that,

Determinant of 3 × 3 matrix is given as,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}.\det\begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} - a_{12}.\det\begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix} + a_{13}.\det\begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

$$\Rightarrow \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}(a_{22} \times a_{33} - a_{23} \times a_{32}) - a_{12}(a_{21} \times a_{33} - a_{23} \times a_{31}) + a_{13}(a_{21} \times a_{32} - a_{22} \times a_{31})$$

So,

$$\begin{vmatrix} a^2 & a & 1 \\ \cos nx & \cos(n+1)x & \cos(n+2)x \\ \sin nx & \sin(n+1)x & \sin(n+2)x \end{vmatrix} = a^2.\det\begin{bmatrix} \cos(n+1)x & \cos(n+2)x \\ \sin(n+1)x & \sin(n+2)x \end{bmatrix} - a.\det\begin{bmatrix} \cos nx & \cos(n+2)x \\ \sin nx & \sin(n+2)x \end{bmatrix} + \det\begin{bmatrix} \cos nx & \cos(n+1)x \\ \sin nx & \sin(n+1)x \end{bmatrix}$$

$$\Rightarrow \begin{vmatrix} a^2 & a & 1 \\ \cos nx & \cos(n+1)x & \cos(n+2)x \\ \sin nx & \sin(n+1)x & \sin(n+2)x \end{vmatrix} = a^2(\cos(n+1)x \times \sin(n+2)x - \cos(n+2)x \times \sin(n+1)x) - a(\cos nx \times \sin(n+2)x - \cos(n+2)x \times \sin nx) + (\cos nx \times \sin(n+1)x - \cos(n+1)x \times \sin nx)$$

By trigonometric identity, we have

sin (α – β) = sin α cos β – cos α sin β

So, we can write

$$\Rightarrow \begin{vmatrix} a^2 & a & 1 \\ \cos nx & \cos(n+1)x & \cos(n+2)x \\ \sin nx & \sin(n+1)x & \sin(n+2)x \end{vmatrix} = a^2 \sin\big((n+2)x - (n+1)x\big) - a\sin\big((n+2)x - nx\big) + \sin\big((n+1)x - nx\big)$$

$$\Rightarrow \begin{vmatrix} a^2 & a & 1 \\ \cos nx & \cos(n+1)x & \cos(n+2)x \\ \sin nx & \sin(n+1)x & \sin(n+2)x \end{vmatrix} = a^2 \sin(nx + 2x - nx - x) - a\sin(nx + 2x - nx) + \sin(nx + x - nx)$$

$$\Rightarrow \begin{vmatrix} a^2 & a & 1 \\ \cos nx & \cos(n+1)x & \cos(n+2)x \\ \sin nx & \sin(n+1)x & \sin(n+2)x \end{vmatrix} = a^2 \sin x - a\sin 2x + \sin x$$

Note that, the result has ‘a’ as well as ‘x’, but doesn’t contain ‘n’.

Thus, the determinant is independent of n.

7. Question

Mark the correct alternative in the following:

If $\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} 1 & bc & a \\ 1 & ca & b \\ 1 & ab & c \end{vmatrix}$, then

A. $\Delta_1 + \Delta_2 = 0$

B. $\Delta_1 + 2\Delta_2 = 0$

C. $\Delta_1 = \Delta_2$

D. none of these

Answer

We are given that,

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} \text{ and } \Delta_2 = \begin{vmatrix} 1 & bc & a \\ 1 & ca & b \\ 1 & ab & c \end{vmatrix}$$

Let us find the determinants Δ_1 and Δ_2.

We know that,

Determinant of 3 × 3 matrix is given as,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}.\det\begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} - a_{12}.\det\begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix} + a_{13}.\det\begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

$$\Rightarrow \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}(a_{22} \times a_{33} - a_{23} \times a_{32}) - a_{12}(a_{21} \times a_{33} - a_{23} \times a_{31}) + a_{13}(a_{21} \times a_{32} - a_{22} \times a_{31})$$

So,

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

$$\Rightarrow \Delta_1 = \det\begin{bmatrix} b & c \\ b^2 & c^2 \end{bmatrix} - \det\begin{bmatrix} a & c \\ a^2 & c^2 \end{bmatrix} + \det\begin{bmatrix} a & b \\ a^2 & b^2 \end{bmatrix}$$

$\Rightarrow \Delta_1 = (b \times c^2 - c \times b^2) - (a \times c^2 - c \times a^2) + (a \times b^2 - b \times a^2)$

$\Rightarrow \Delta_1 = bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b$ ...(i)

Also,

$$\Rightarrow \Delta_2 = \begin{vmatrix} 1 & bc & a \\ 1 & ca & b \\ 1 & ab & c \end{vmatrix}$$

$$\Rightarrow \Delta_2 = \det\begin{bmatrix} ca & b \\ ab & c \end{bmatrix} - bc.\det\begin{bmatrix} 1 & b \\ 1 & c \end{bmatrix} + a.\det\begin{bmatrix} 1 & ca \\ 1 & ab \end{bmatrix}$$

$\Rightarrow \Delta_2 = (ca \times c - b \times ab) - bc(1 \times c - b \times 1) + a(1 \times ab - ca \times 1)$

$\Rightarrow \Delta_2 = ac^2 - ab^2 - bc(c - b) + a(ab - ac)$

$\Rightarrow \Delta_2 = ac^2 - ab^2 - bc^2 + b^2c + a^2b - a^2c$ ...(ii)

Checking Option (A).

Adding Δ_1 and Δ_2 by using values from (i) and (ii),

$\Delta_1 + \Delta_2 = (bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b) + (ac^2 - ab^2 - bc^2 + b^2c + a^2b - a^2c)$

$\Rightarrow \Delta_1 + \Delta_2 = bc^2 - bc^2 - b^2c + b^2c - ac^2 + ac^2 + ab^2 - ab^2 - a^2b + a^2b$

$\Rightarrow \Delta_1 + \Delta_2 = 0$

Thus, option (A) is correct.

Checking Option (B).

Multiplying 2 by (ii),

$2\Delta_2 = 2(ac^2 - ab^2 - bc^2 + b^2c + a^2b - a^2c)$

$\Rightarrow 2\Delta_2 = 2ac^2 - 2ab^2 - 2bc^2 + 2b^2c + 2a^2b - 2a^2c$ ...(iii)

Then, adding $2\Delta_2$ with Δ_1,

$\Delta_1 + 2\Delta_2 = (bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b) + (2ac^2 - 2ab^2 - 2bc^2 + 2b^2c + 2a^2b - 2a^2c)$

$\Rightarrow \Delta_1 + 2\Delta_2 = bc^2 - 2bc^2 - b^2c + 2b^2c - ac^2 + 2ac^2 + ab^2 - 2ab^2 - a^2b + 2a^2b$

$\Rightarrow \Delta_1 + 2\Delta_2 = -bc^2 + b^2c + ac^2 - ab^2 + a^2b$

$\Rightarrow \Delta_1 + 2\Delta_2 \neq 0$

Thus, option (B) is not correct.

Checking option (C).

Obviously, $\Delta_1 \neq \Delta_2$

Since, by (i) and (ii), we can notice Δ_1 and Δ_2 have different values.

Thus, option (C) is not correct.

8. Question

Mark the correct alternative in the following:

If $D_k = \begin{vmatrix} 1 & n & n \\ 2k & n^2+n+2 & n^2+n \\ 2k-1 & n^2 & n^2+n+2 \end{vmatrix}$ and $\sum_{k=1}^{n} D_k = 48$, then n equals

A. 4

B. 6

C. 8

D. none of these

Answer

We are given that,

$$\begin{vmatrix} x^2+3x & x-1 & x+3 \\ x+1 & -2x & x-4 \\ x-3 & x+4 & 3x \end{vmatrix} = ax^4 + bx^3 + cx^2 + dx + e$$

We need to find the value of e.

We know that,

Determinant of 3 × 3 matrix is given as,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}.\det\begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} - a_{12}.\det\begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix} + a_{13}.\det\begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

9. Question

Mark the correct alternative in the following:

Let $\begin{vmatrix} x^2+3x & x-1 & x+3 \\ x+1 & -2x & x-4 \\ x-3 & x+4 & 3x \end{vmatrix} = ax^4+bx^3+cx^2+dx+e$ be an identify in x, were a, b, c, d, e are independent of x. Then the value of e is

A. 4

B. 0

C. 1

D. none of these

Answer

We are given that,

$$\begin{vmatrix} x^2+3x & x-1 & x+3 \\ x+1 & -2x & x-4 \\ x-3 & x+4 & 3x \end{vmatrix} = ax^4+bx^3+cx^2+dx+e$$

We need to find the value of e.

We know that,

Determinant of 3 × 3 matrix is given as,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}.\det\begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} - a_{12}.\det\begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix} + a_{13}.\det\begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

$$\Rightarrow \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}(a_{22}\times a_{33} - a_{23}\times a_{32}) - a_{12}(a_{21}\times a_{33} - a_{23}\times a_{31}) + a_{13}(a_{21}\times a_{32} - a_{22}\times a_{31})$$

So,

$$\begin{vmatrix} x^2+3x & x-1 & x+3 \\ x+1 & -2x & x-4 \\ x-3 & x+4 & 3x \end{vmatrix} = ax^4+bx^3+cx^2+dx+e$$

$$\Rightarrow (x^2+3x).\det\begin{bmatrix} -2x & x-4 \\ x+4 & 3x \end{bmatrix} - (x-1).\det\begin{bmatrix} x+1 & x-4 \\ x-3 & 3x \end{bmatrix} + (x+3).\det\begin{bmatrix} x+1 & -2x \\ x-3 & x+4 \end{bmatrix} = ax^4+bx^3+cx^2+dx+e$$

⇒ $(x^2 + 3x)[-2x \times 3x - (x - 4)(x + 4)] - (x - 1)[(x + 1) \times 3x - (x - 4)(x - 3)] + (x + 3)[(x + 1)(x + 4) - (-2x)(x - 3)] = ax^4 + bx^3 + cx^2 + dx + e$

⇒ $(x^2 + 3x)[-6x - (x^2 - 16)] - (x - 1)[3x(x + 1) - (x^2 - 3x - 4x + 12)] + (x + 3)[x^2 + x + 4x + 4 + 2x(x - 3)] = ax^4 + bx^3 + cx^2 + dx + e$

⇒ $(x^2 + 3x)[-6x - x^2 + 16] - (x - 1)[3x^2 + 3x - x^2 + 7x - 12] + (x + 3)[x^2 + 5x + 4 + 2x^2 - 6x] = ax^4 + bx^3 + cx^2 + dx + e$

⇒ $-x^4 - 6x^3 + 16x^2 - 3x^3 - 18x^2 + 48x - (x - 1)[2x^2 + 10x - 12] + (x + 3)[3x^2 - x + 4] = ax^4 + bx^3 + cx^2 + dx + e$

⇒ $-x^4 - 9x^3 - 2x^2 + 48x - (2x^3 - 2x^2 + 10x^2 - 10x - 12x + 12) + 3x^3 + 9x^2 - x^2 - 3x + 4x + 12 = ax^4 + bx^3 + cx^2 + dx + e$

⇒ $-x^4 - 9x^3 - 2x^2 + 48x - 2x^3 + 2x^2 - 10x^2 + 10x + 12x - 12 + 3x^3 + 9x^2 - x^2 - 3x + 4x + 12 = ax^4 + bx^3 + cx^2 + dx + e$

⇒ $-x^4 - 9x^3 - 2x^3 + 3x^3 - 2x^2 + 2x^2 + 9x^2 - x^2 + 48x + 10x + 12x - 3x + 4x - 12 + 12 = ax^4 + bx^3 + cx^2 + dx + e$

⇒ $-x^4 - 8x^3 + 8x^2 + 23x + 0 = ax^4 + bx^3 + cx^2 + dx + e$

Comparing left hand side and right-hand side of the equation, we get

$e = 0$

Thus, $e = 0$.

10. Question

Mark the correct alternative in the following:

Using the factor theorem it is found that a + b, b + c and c + a are three factors of the determinant $\begin{vmatrix} -2a & a+b & a+c \\ b+a & -2b & b+c \\ c+a & c+b & -2c \end{vmatrix}$. The other factor in the value of the determinant is

A. 4

B. 2

C. a + b + c

D. none of these

Answer

$$\begin{vmatrix} -2a & a+b & c+a \\ a+b & -2b & b+c \\ c+a & b+c & -2c \end{vmatrix} = k(a+b)(b+c)(c+a)$$

Let assume a=0, b=1, c=2

$$\begin{vmatrix} 0 & 1 & 2 \\ 1 & -2 & 3 \\ 2 & 3 & -2 \end{vmatrix} = k(a+b)(b+c)(c+a)$$

$$\begin{vmatrix} 0 & 1 & 2 \\ 1 & -2 & 3 \\ 2 & 3 & -2 \end{vmatrix} = k(0+1)(1+2)(2+0)$$

Now expending around colume1

0-1(-4-6)+2(3+4)=k(1)(3)(2)

6k=24

K=4

11. Question

Mark the correct alternative in the following:

If a, b, c are distinct then the value of x satisfying $\begin{vmatrix} 0 & x^2-a & x^3-b \\ x^2+a & 0 & x^2+c \\ x^4+b & x-c & 0 \end{vmatrix} = 0$ is

A. c

B. a

C. b

D. 0

Answer

$\Delta = \begin{vmatrix} 0 & x^2-a & x^3-b \\ x^2+a & 0 & x^2+c \\ x^4+b & x-c & 0 \end{vmatrix}$

$\Delta = \begin{vmatrix} 0 & x^2+a & x^4+b \\ x^2-a & 0 & x-c \\ x^3-b & x^2+c & 0 \end{vmatrix}$

$2\Delta = \begin{vmatrix} 0 & 2x^2 & x^4+x^3 \\ 2x^2 & 0 & x^2+x \\ x^3+x^4 & x^2+x & 0 \end{vmatrix}$

$\Delta = 0$ (this is possible when x=0)

12. Question

Mark the correct alternative in the following:

If the determinant $\begin{vmatrix} a & b & 2a\alpha+3b \\ b & c & 2b\alpha+3c \\ 2a\alpha+3b & 2b\alpha+3c & 0 \end{vmatrix} = 0$, then

A. a, b, c are in H.P.

B. α is a root of $4ax^2 + 12bx + 9c = 0$ or, a, b, c are in G.P.

C. a, b, c are in G.P. only

D. a, b, c are in A.P.

Answer

expend the determinats

$a[-(2b\alpha+3c)^2]-b[-(2b\alpha+3c)(2a\alpha+3b)]+(2a\alpha+3b)[b(2b\alpha+3c)-c(2a\alpha+3b)]=0$

-a(2bα+3c)^2 + b(2bα+3c)(2aα+3b)+(2aα+3b)[2b^2 α+3bc-3bc-2acα]=0

(2bα+3c) [-2abα-3ac+2abα+3b^2]+ (2aα+3b)(2α)(b^2 -ac)=0

(2bα+3c) [-3ac +3b^2]+ (2aα+3b)(2α)(b^2-ac)=0

(b^2 – ac)[4aα^2 + 12bα + ac] = 0=

CASE1→(b^2 -ac)=0

b^2 =ac {abc are in Gp}

CASE2→(4aα^2 +12bα+ac)=0 {Whose one root is α}

13. Question

Mark the correct alternative in the following:

If ω is a non-real cube root of unity and n is not a multiple of 3, then $\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^{2n} & 1 & \omega^n \\ \omega^n & \omega^{2n} & 1 \end{vmatrix}$ is equal to

A. 0

B. ω

C. ω^2

D. 1

Answer

Assume that n=2(not multiple of 3)

$\Delta = \begin{vmatrix} 1 & w^2 & w^4 \\ w^4 & 1 & w^2 \\ w^2 & w^4 & 1 \end{vmatrix}$

$\Delta = \begin{vmatrix} 1 & w^2 & w \\ w & 1 & w^2 \\ w^2 & w & 1 \end{vmatrix}$ expend the determinant

$\Delta=1(1-w^3)-w^2(w-w^4)+w(w^2-w^2)$

$\Delta=1-w^3-w^3+w^6+w^3-w^3$

Δ=0

14. Question

Mark the correct alternative in the following:

If $A_r = \begin{vmatrix} 1 & r & 2^r \\ 2 & n & n^2 \\ n & \frac{n(n+1)}{2} & 2^{n+1} \end{vmatrix}$, then the value of $\sum_{r=1}^{n} A_r$ is

A. n

B. 2n

C. -2n

D. n^2

Answer

$\sum_{r=1}^{n} A_r = \begin{vmatrix} 1 & \sum_{r=1}^{n} r & \sum_{r=1}^{n} 2^r \\ 2 & n & n^2 \\ n & \frac{n(n+1)}{2} & 2^{n+1} \end{vmatrix}$

$\sum_{r=1}^{n} A_r = \begin{vmatrix} 1 & \frac{n(n+1)}{2} & 2(2^n-1) \\ 2 & n & n^2 \\ n & \frac{n(n+1)}{2} & 2^{n+1} \end{vmatrix}$ assume (n)=1

$\sum_{r=1}^{n} A_r = \begin{vmatrix} 1 & 1 & 2) \\ 2 & 1 & 1 \\ 1 & 1 & 4 \end{vmatrix}$

1(4-1)-1(8-1) +2(2-1)=-2

Answer=c(-2n)

15. Question

Mark the correct alternative in the following:

If a > 0 and discriminant of $ax^2 + 2bx + c$ is negative, then $\Delta = \begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix}$ is

A. positive

B. $(ac - b^2)(ax^2 + 2bx + c)$

C. negative

D. 0

Answer

discriminant of $ax^2 + 2bx + c = 0$

$4b^2 - 4ac < 0$ and $ax^2 + 2bx + c > 0$

$$\Delta = \begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix}$$

$R_3 \rightarrow R_3 - X R_1 - R_2$

$$\Delta = \begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ 0 & 0 & -(2ax + 2bx + c) \end{vmatrix}$$

$-(2ax + 2bx + c)(-b^2 + ac) < 0$

16. Question

Mark the correct alternative in the following:

The value of $\begin{vmatrix} 5^2 & 5^3 & 5^4 \\ 5^3 & 5^4 & 5^5 \\ 5^4 & 5^5 & 5^6 \end{vmatrix}$ is

A. 5^2

B. 0

C. 5^{13}

D. 5^9

Answer

$$\Delta = \begin{vmatrix} 5^2 & 5^3 & 5^4 \\ 5^3 & 5^4 & 5^5 \\ 5^4 & 5^5 & 5^6 \end{vmatrix}$$

$$\Delta = 5^9 \begin{vmatrix} 1 & 5 & 5^2 \\ 1 & 5 & 5^2 \\ 1 & 5 & 5^2 \end{vmatrix}$$

$$\Delta = 5^9 5^3 \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

Δ=0

17. Question

Mark the correct alternative in the following:

$$\begin{vmatrix} \log_3 512 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} \times \begin{vmatrix} \log_2 3 & \log_8 3 \\ \log_3 4 & \log_3 4 \end{vmatrix} =$$

A. 7

B. 10

C. 13

D. 17

Answer

$$= \begin{vmatrix} \log_3 512 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} \times \begin{vmatrix} \log_2 3 & \log_8 3 \\ \log_3 4 & \log_3 4 \end{vmatrix}$$

$$= \begin{vmatrix} 9\log_3 2 & \log_4 3 \\ 3\log_3 2 & 2\log_4 3 \end{vmatrix} \times \begin{vmatrix} \log_2 3 & \frac{1}{3}\log_2 3 \\ 2\log_3 2 & 2\log_3 2 \end{vmatrix}$$

$$= \log_3 2 \log_4 3 \log_2 3 \log_3 2 \begin{vmatrix} 9 & 1 \\ 3 & 2 \end{vmatrix} \times \begin{vmatrix} 1 & \frac{1}{3} \\ 2 & 2 \end{vmatrix}$$

$$= \frac{1}{2} \begin{vmatrix} 9 & 1 \\ 3 & 2 \end{vmatrix} \times \begin{vmatrix} 1 & \frac{1}{3} \\ 2 & 2 \end{vmatrix}$$

$$= \frac{1}{2} \begin{vmatrix} 9+2 & 3+2 \\ 3+4 & 1+4 \end{vmatrix}$$

$$= \frac{1}{2} \begin{vmatrix} 11 & 5 \\ 7 & 5 \end{vmatrix}$$

$$= \frac{1}{2}(55 - 35)$$

=10

18. Question

Mark the correct alternative in the following:

If a, b, c are in A.P., then the determinant $\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$

A. 0

B. 1

C. x

D. 2x

Answer

$$\Delta=\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$$

{a + c=2b}

$R_1 \to R_1 - R_2$

$R_2 \to R_2 - R_3$

$$\Delta=\begin{vmatrix} -1 & -1 & a-c \\ -1 & -1 & a-c \\ x+4 & x+5 & x+2c \end{vmatrix}$$

$R_1 \to R_1 - R_2$

$$\Delta=\begin{vmatrix} 0 & 0 & 0 \\ -1 & -1 & a-c \\ x+4 & x+5 & x+2c \end{vmatrix}$$

$\Delta=0$

19. Question

Mark the correct alternative in the following:

If A + B + C = π, then the value of $\begin{vmatrix} \sin(A+B+C) & \sin(A+C) & \cos C \\ -\sin B & 0 & \tan A \\ \cos(A+B) & \tan(B+C) & 0 \end{vmatrix}$ is equal to

A. 0

B. 1

C. 2sin B tan A cos C

D. none of these

Answer

$$\Delta=\begin{vmatrix} \sin\pi & \sin\pi-B & \cos C \\ -\sin B & 0 & \tan A \\ \cos\pi-C & \tan\pi-A & 0 \end{vmatrix}$$

$$\Delta=\begin{vmatrix} \sin\pi & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ -\cos C & -\tan A & 0 \end{vmatrix}$$

ON TRANSPOSING

$$\Delta=\begin{vmatrix} \sin\pi & -\sin B & -\cos C \\ \sin B & 0 & -\tan A \\ \cos C & \tan A & 0 \end{vmatrix}$$

$$2\Delta=\begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

$\Delta=0$

20. Question

Mark the correct alternative in the following:

The number of distinct real roots of $\begin{vmatrix} \operatorname{cosec} x & \sec x & \sec x \\ \sec x & \operatorname{cosec} x & \sec x \\ \sec x & \sec x & \operatorname{cosec} x \end{vmatrix} = 0$ lies in the interval $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ is

A. 1

B. 2

C. 3

D. 0

Answer

$$\Delta = \begin{vmatrix} \csc x & \sec x & \sec x \\ \sec x & \csc x & \sec x \\ \sec x & \sec x & \csc x \end{vmatrix}$$

$c_1 \to c_1 + c_2 + c_3$

$$\Delta = \begin{vmatrix} \csc x + 2\sec x & \sec x & \sec x \\ 2\sec x + \csc x & \csc x & \sec x \\ 2\sec x + \csc x & \sec x & \csc x \end{vmatrix}$$

$$\Delta = (\csc x + 2\sec x)\begin{vmatrix} 1 & \sec x & \sec x \\ 1 & \csc x & \sec x \\ 1 & \sec x & \csc x \end{vmatrix}$$

Δ=(csc x+2 sec x)[(csc x-sec x)^2]

Case1: (csc x+2 sec x)=0

$\tan x = -\frac{1}{2}$ (1$^{\text{st}}$ real root)

Case: (csc x-sec x)^2 =0

Tan x=1 (2$^{\text{nd}}$ real root)

21. Question

Mark the correct alternative in the following:

Let $A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$, where 0 ≤ θ ≤ 2π. Then,

A. Det (A) = 0

B. Det (A) ∈ (2, ∞)

C. Det (A) ∈ (2, 4)

D. Det (A) ∈ [2, 4]

Answer

$$A = \begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix}$$

$R_1 \to R_1 + R_3$

$$A = \begin{vmatrix} 0 & 0 & 2 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix}$$

$A=2[(\sin\theta)^2+1]$ $0\le(\sin\theta)^2\le1$

$A\in 2[1,2]$

$A\in[2,4]$

22. Question

Mark the correct alternative in the following:

If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix}=\begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, then x =

A. 3

B. ± 3

C. ± 6

D. 6

Answer

$\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix}=\begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$

$2x^2-40=18+14$

$x=\pm 6$

23. Question

Mark the correct alternative in the following:

If $f(x)=\begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix}$, then

A. f(a) = 0

B. 3bc

C. $a^3+b^3+c^3-3abc$

D. none of these

Answer

$$f(x)=\begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix}$$

ON TRANSPOSING

$$f(x)=\begin{vmatrix} 0 & x+a & x+b \\ x-a & 0 & x+c \\ x-b & x-c & 0 \end{vmatrix}$$

$$2f(x)=\begin{vmatrix} 0 & 2x & 2x \\ 2x & 0 & 2x \\ 2x & 2x & 0 \end{vmatrix}$$

$$2f(x)=8\begin{vmatrix} 0 & x & x \\ x & 0 & x \\ x & x & 0 \end{vmatrix}$$

$$f(x)=4[-x(-x^2)+x(x^2-0)]$$

$f(x) = 8x^3$

24. Question

Mark the correct alternative in the following:

The value of the determinant $\begin{vmatrix} a-b & b+c & a \\ b-a & c+a & b \\ c-a & a+b & c \end{vmatrix}$ is

A. $a^3 + b^3 + c^3$

B. 3bc

C. $a^3 + b^3 + c^3 - 3abc$

D. none of these

Answer

assume a=1,b=2, c=3 (put in determinant)

$$\Delta = \begin{vmatrix} -1 & 5 & 1 \\ 1 & 4 & 2 \\ 2 & 3 & 3 \end{vmatrix}$$

Δ=[-1(12-6)-5(3-4)+1(3-6)]

Δ=-4

put a=1,b=2, c=3 in option A,B,C,D

ANSWER=D(none of these)

25. Question

Mark the correct alternative in the following:

If x, y, z are different from zero and $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} = 0$, then the value of $x^{-1} + y^{-1} + z^{-1}$ is

A. xyz

B. $x^{-1}y^{-1}z^{-1}$

C. –x –y – z

D. –1

Answer

$$\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} = 0$$

$C_1 \to C_1 - C_2$

$C_3 \to C_3 - C_2$

$$\begin{vmatrix} x & 1 & 0 \\ -y & 1+y & -y \\ 0 & 1 & z \end{vmatrix} = 0$$

x[(1+y)z+ y]-1[-yz]=0

xz + xyz+ xy+yz =0 (divide by xyz in both side)

$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=-1$

26. Question

Mark the correct alternative in the following:

The determinant $\begin{vmatrix} b^2-ab & b-c & bc-ac \\ ab-a^2 & a-b & a^2-ab \\ bc-ca & c-a & ab-a^2 \end{vmatrix}$ equals

A. abc(b – c)(c – a)(a – b)

B. (b – c)(c – a)(a – b)

C. (a + b + c)(b – c)(c – a)(a – b)

D. none of these

Answer

assume a=1,b=2, c=3 (put in determinant)

$\Delta=\begin{vmatrix} 2 & -1 & 3 \\ 1 & -1 & -1 \\ 3 & 2 & 1 \end{vmatrix}$

$\Delta=[2(-1+2)+1(1+3)+3(2+3)]$

$\Delta=21$

put a=1,b=2, c=3 in option A,B,C,D

ANSWER=D(none of these)

27. Question

Mark the correct alternative in the following:

If x, y ∈ R, then the determinant $\Delta=\begin{vmatrix} \cos x & -\sin x & 1 \\ \sin x & \cos x & 1 \\ \cos(x+y) & -\sin(x+y) & 0 \end{vmatrix}$ lies in the interval

A. $\left[-\sqrt{2},\sqrt{2}\right]$ B. [–1, 1]

C. $\left[-\sqrt{2},1\right]$ D. $\left[-1,-\sqrt{2}\right]$

Answer

$\Delta=\begin{vmatrix} \cos x & -\sin x & 1 \\ \sin x & \cos x & 1 \\ \cos(x+y) & -(\sin(x+y) & 0 \end{vmatrix}$

$\cos(x+y)=\cos x\cos y-\sin x\sin y$

$\sin(x+y)=\sin x\cos y+\cos x\sin y$

$\Delta=\begin{vmatrix} \cos x & -\sin x & 1 \\ \sin x & \cos x & 1 \\ \cos x\cos y-\sin x\sin y & -(\sin x\cos y+\cos x\sin y) & 0 \end{vmatrix}$

$R_3\rightarrow R_{3-}\cos y\,R_1+\sin y\;R_2$

$$\Delta=\begin{vmatrix}\cos x & -\sin x & 1\\ \sin x & \cos x & 1\\ 0 & 0 & \sin y-\cos y\end{vmatrix}$$

$\Delta = (\sin y - \cos y)[(\cos x)^2 + (\sin x)^2]$

$= (\sin y - \cos y)$

$= -(\cos y - \sin y)$

$$\Delta=-\sqrt{2}[(\frac{1}{\sqrt{2}}\cos y-\frac{1}{\sqrt{2}}\sin y)]$$

$$\Delta=-\sqrt{2}[(\sin\frac{\pi}{4}\cos y-\cos\frac{\pi}{4}\sin y)]$$

$$\Delta=-\sqrt{2}[\sin(\frac{\pi}{4}-y)] \quad -1\le\sin(\frac{\pi}{4}-y)\ \le 1$$

$\Delta\in\left[-\sqrt{2},\sqrt{2}\right]$

28. Question

Mark the correct alternative in the following:

The maximum value of $\Delta=\begin{vmatrix}1 & 1 & 1\\ 1 & 1+\sin\theta & 1\\ 1+\cos\theta & 1 & 1\end{vmatrix}$ is (θ is real)

A. $\frac{1}{2}$

B. $\frac{\sqrt{3}}{2}$

C. $\sqrt{2}$

D. $-\frac{\sqrt{3}}{2}$

Answer

$$\Delta=\begin{vmatrix}1 & 1 & 1\\ 1 & 1+\sin\theta & 1\\ 1+\cos\theta & 1 & 1\end{vmatrix}$$

$c_1\to c_1-c_3$

$$\Delta=\begin{vmatrix}0 & 1 & 1\\ 0 & 1+\sin\theta & 1\\ \cos\theta & 1 & 1\end{vmatrix}$$

$\Delta=\cos\theta\,(1-1-\sin\theta)$

$\Delta=-\cos\theta\sin\theta$

$$\Delta=-\frac{1}{2}\sin 2\theta$$

$-1\le\sin 2\theta\le 1$

$\Delta=\frac{1}{2}\,[\theta=-\frac{\pi}{4}]$

29. Question

Mark the correct alternative in the following:

The value of the determinant $\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix}$ is

A. $9x^2(x + y)$

B. $9y^2(x + y)$

C. $3y^2(x + y)$

D. $7x^2(x + y)$

Answer

$$\Delta = \begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix}$$

$R_1 \to R_1 - R_3$

$R_2 \to R_2 - R_3$

$$\Delta = \begin{vmatrix} -y & -y & 2y \\ y & -2y & y \\ x+y & x+2y & x \end{vmatrix}$$

$$\Delta = y^2 \begin{vmatrix} -1 & -1 & 2 \\ 1 & -2 & 1 \\ x+y & x+2y & x \end{vmatrix}$$

$R_1 \to R_1 + R_2$

$$\Delta = y^2 \begin{vmatrix} 0 & -3 & 3 \\ 1 & -2 & 1 \\ x+y & x+2y & x \end{vmatrix}$$

$\Delta = y^2[-1(-3x - 3x - 6y) + (x+y)(-3+6)]$

$\Delta = y^2[6(x+y) + 3(x+y)]$

$\Delta = 9y^2(x+y)$

30. Question

Mark the correct alternative in the following:

Let $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x & 2x \\ \sin x & x & x \end{vmatrix}$, then $\lim\limits_{x\to 0} \frac{f(x)}{x^2}$ is equal to

A. 0

B. -1

C. 2

D. 3

Answer

$f(x) = x[(-x\cos x) + \sin x]$

$f(x) = (-x^2\cos x) + x\sin x$

$$\lim_{x\to 0}\frac{f(x)}{x^2}=\lim_{x\to 0}\frac{-x^2\cos(x)+x\sin x}{x^2}$$

$$\lim_{x\to 0}\frac{-x^2\cos(x)}{x^2}+\lim_{x\to 0}\frac{x\sin x}{x^2}$$

$$\lim_{x\to 0}-\cos x=-1$$

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

ANSWER= -1+1=0

31. Question

Mark the correct alternative in the following:

There are two values of a which makes the determinant $\Delta=\begin{vmatrix}1 & -2 & 5\\ 2 & a & -1\\ 0 & 4 & 2a\end{vmatrix}$ equal to 86. The sum of these two values is

A. 4

B. 5

C. –4

D. 9

Answer

$$\Delta=\begin{vmatrix}1 & -2 & 5\\ 2 & a & -1\\ 0 & 4 & 2a\end{vmatrix}$$

$$\Delta=(2a^2+4)+2(4a)+40$$

$$43=a^2+4a+22$$

Sum of roots = $-\frac{b}{a}$ [b=1and a=1]

Sum of roots= -4

32. Question

Mark the correct alternative in the following:

If $\begin{vmatrix}a & p & x\\ b & q & y\\ c & r & z\end{vmatrix}=16$, t hen the value of $\begin{vmatrix}p+q & a+x & a+p\\ q+y & b+y & b+q\\ x+z & c+z & c+r\end{vmatrix}$ is

A. 4

B. 8

C. 16

D. 32

Answer

$$\begin{vmatrix}p+q & a+x & a+p\\ q+y & b+y & b+q\\ x+z & c+z & c+r\end{vmatrix}$$

$c_1 \to c_1 + c_2 + c_3$

$$\begin{vmatrix} 2a+2p+q+x & a+x & a+p \\ 2b+2q+y+b & b+y & b+q \\ 2c+x+2z+r & c+z & c+r \end{vmatrix}$$

$$2\begin{vmatrix} a & x & p \\ b & y & q \\ c & z & r \end{vmatrix} + \begin{vmatrix} 2p+q+x & a & a \\ 2q+y+b & b & b \\ x+2z+r & c & c \end{vmatrix}$$

$$2\begin{vmatrix} a & x & p \\ b & y & q \\ c & z & r \end{vmatrix} + 0$$

2×16=32

33. Question

Mark the correct alternative in the following:

The value of $\begin{vmatrix} 1 & 1 & 1 \\ {}^{n}C_1 & {}^{n+2}C_1 & {}^{n+4}C_1 \\ {}^{n}C_2 & {}^{n+2}C_2 & {}^{n+4}C_2 \end{vmatrix}$ is

A. 2

B. 4

C. 8

D. n^2

Answer

$$\begin{vmatrix} 1 & 1 & 1 \\ {}^{n}C_1 & {}^{n+2}C_1 & {}^{n+4}C_1 \\ {}^{n}C_2 & {}^{n+2}C_2 & {}^{n+4}C_2 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ n & n+2 & n+4 \\ n^2-n & n^2+3n+2 & n^2+7n+12 \end{vmatrix}$$

$c_1 \to c_1 - c_2$

$c_2 \to c_2 - c_3$

$$\Delta = \begin{vmatrix} 0 & 0 & 1 \\ -2 & -2 & n+4 \\ -4n-2 & -4n-10 & n^2+7n+12 \end{vmatrix}$$

Δ= 1/2 [8n+20-8n-4]

Δ=8

Very short answer

1. Question

If A is a singular matrix, then write the value of |A|.

Answer

Since a singular matrix is a matrix whose determinant is 0, Therefore the determinant of A is 0.

2. Question

For what value of x, the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular?

Answer

$A = \begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$

Hence $|A| = \begin{vmatrix} 5-x & x+1 \\ 2 & 4 \end{vmatrix}$

=(5-x)×4-(x+1)×2 (Expanding along R_1)

|A|=18-6x

For A to be a singular matrix, |A| has to be 0.

Therefore, 18-6x=0 or x=3.

3. Question

Write the value of the determinant $\begin{vmatrix} 2 & 3 & 4 \\ 2x & 3x & 4x \\ 5 & 6 & 8 \end{vmatrix}$.

Answer

Let $\Delta = \begin{vmatrix} 2 & 3 & 4 \\ 2x & 3x & 4x \\ 5 & 6 & 8 \end{vmatrix}$

Using the property that if the equimultiples of corresponding elements of other rows (or columns) are added to every element of any row (or column) of a determinant, then the value of determinant remains the same

Using row transformation, $R_2 \rightarrow R_2 - xR_1$

$\Delta = \begin{vmatrix} 2 & 3 & 4 \\ 2x-2x & 3x-3x & 4x-4x \\ 5 & 6 & 8 \end{vmatrix} = \begin{vmatrix} 2 & 3 & 4 \\ 0 & 0 & 0 \\ 5 & 6 & 8 \end{vmatrix}$

Using the property that if all elements of any row or column of a determinant are 0, then the value of determinant is 0.

Since R_2 has all elements 0, therefore Δ=0.

4. Question

State whether the matrix $\begin{bmatrix} 2 & 3 \\ 6 & 4 \end{bmatrix}$ is singular or non-singular.

Answer

Let $A = \begin{bmatrix} 2 & 3 \\ 6 & 4 \end{bmatrix}$

Then $|A| = \begin{vmatrix} 2 & 3 \\ 6 & 4 \end{vmatrix}$

=2×4-3×6

=-10 (Expanding along R_1)

Since |A|≠0, therefore A is a non-singular matrix.

5. Question

Find the value of the determinant $\begin{vmatrix} 4200 & 4201 \\ 4202 & 4203 \end{vmatrix}$.

Answer

Let $\Delta = \begin{vmatrix} 4200 & 4201 \\ 4202 & 4203 \end{vmatrix}$

$= \begin{vmatrix} 0+4200 & 1+4200 \\ 2+4200 & 3+4200 \end{vmatrix}$

Using the property that if some or all elements of a row or column of a determinant are expressed as the sum of two (or more) terms, then the determinant can be expressed as the sum of two (or more) determinants.

We get, $\Delta = \begin{vmatrix} 0 & 1 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 4200 & 4200 \\ 4200 & 4200 \end{vmatrix}$

Using the property that If any two rows (or columns) of a determinant are identical (all corresponding elements are same), then the value of the determinant is zero.

Hence, $\Delta = \begin{vmatrix} 0 & 1 \\ 2 & 3 \end{vmatrix} + 0$

$=0\times3-1\times2=-2$ (Expanding along R_1)

6. Question

Find the value of the determinant $\begin{vmatrix} 101 & 102 & 103 \\ 104 & 105 & 106 \\ 107 & 108 & 109 \end{vmatrix}$.

Answer

Let $\Delta = \begin{vmatrix} 101 & 102 & 103 \\ 104 & 105 & 106 \\ 107 & 108 & 109 \end{vmatrix}$

$= \begin{vmatrix} 1+100 & 2+100 & 3+100 \\ 4+100 & 5+100 & 6+100 \\ 7+100 & 8+100 & 9+100 \end{vmatrix}$

Using the property that if some or all elements of a row or column of a determinant are expressed as the sum of two (or more) terms, then the determinant can be expressed as the sum of two (or more) determinants.

We get, $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} + \begin{vmatrix} 100 & 100 & 100 \\ 100 & 100 & 100 \\ 100 & 100 & 100 \end{vmatrix}$

Using the property that If any two rows (or columns) of a determinant are identical (all corresponding elements are same), then the value of the determinant is zero.

We get, $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} + 0$

$= \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$

Using the property that if the equimultiples of corresponding elements of other rows (or columns) are added to every element of any row (or column) of a determinant, then the value of determinant remains the same.

Using row transformation, $R_2 \to R_2 - R_1$ and $R_3 = R_3 - R_1$

We get, $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 4-1 & 5-2 & 6-3 \\ 7-1 & 8-2 & 9-3 \end{vmatrix}$

$= \begin{vmatrix} 1 & 2 & 3 \\ 3 & 3 & 3 \\ 6 & 6 & 6 \end{vmatrix}$

$= \begin{vmatrix} 1 & 2 & 3 \\ 3 & 3 & 3 \\ 2\times 3 & 2\times 3 & 2\times 3 \end{vmatrix}$

Using the property that if each element of a row (or a column) of a determinant is multiplied by a constant k, then its value gets multiplied by k.

Taking out factor 2 from R_3,

We get, $\Delta = 2\begin{vmatrix} 1 & 2 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{vmatrix}$

Using the property that If any two rows (or columns) of a determinant are identical (all corresponding elements are same), then the value of the determinant is zero.

Since, R_2 and R_3 are identical, therefore $\Delta = 0$.

7. Question

Write the value of the determinant $\begin{vmatrix} a & 1 & b+c \\ b & 1 & c+a \\ c & a & a+b \end{vmatrix}$.

Answer

Let $\Delta = \begin{vmatrix} a & 1 & b+c \\ b & 1 & c+a \\ c & a & a+b \end{vmatrix}$

Using the property that if the equimultiples of corresponding elements of other rows (or columns) are added to every element of any row (or column) of a determinant, then the value of determinant remains the same.

Using column transformation, $C_1 \rightarrow C_1 + C_3$

We get, $\Delta = \begin{vmatrix} a+b+c & 1 & b+c \\ b+c+a & 1 & c+a \\ c+a+b & a & a+b \end{vmatrix}$

Using the property that if each element of a row (or a column) of a determinant is multiplied by a constant k, then its value gets multiplied by k.

Taking out factor(a+b+c) from C_1,

We get, $\Delta = (a+b+c) \times \begin{vmatrix} 1 & 1 & b+c \\ 1 & 1 & c+a \\ 1 & a & a+b \end{vmatrix}$

Using column transformation, $C_1 \rightarrow C_1 - C_2$

We get,

$\Delta = (a+b+c) \times \begin{vmatrix} 0 & 1 & b+c \\ 0 & 1 & c+a \\ 1-a & a & a+b \end{vmatrix}$

Expanding along C_1, we get

$\Delta = (a+b+c) \times [(1-a)(c+a-(b+c))] = (1-a)(a-b)(a+b+c)$

8. Question

If $A = \begin{bmatrix} 0 & i \\ i & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$, find the value of |A| + |B|.

Answer

Given that $A = \begin{bmatrix} 0 & i \\ i & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$, we have to find |A|+|B|

Then, $|A| = \begin{vmatrix} 0 & i \\ i & 1 \end{vmatrix}$ and $|B| = \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix}$

|A|=0×1-i×i

$=-i^2$

=1 (Expanding along R_1 and since $i^2=-1$)

|B|=0×1-1×1

=-1 (Expanding along R_1)

|A|+|B|=1-1

0

9. Question

If $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$, find |AB|.

Answer

Given that $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$, we have to find |AB|

Then, $|A| = \begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix}$ and $|B| = \begin{vmatrix} 1 & 0 \\ -1 & 0 \end{vmatrix}$

|A|=1×-1-2×3

=-7 (Expanding along R_1)

|B|=1×0-0×-1

=0 (Expanding along R_1)

Since |AB|=|A||B|,

Therefore |AB|=-7×0=0

10. Question

Evaluate: $\begin{vmatrix} 4785 & 4787 \\ 4789 & 4791 \end{vmatrix}$.

Answer

Let $\Delta = \begin{vmatrix} 4785 & 4787 \\ 4789 & 4791 \end{vmatrix} = \begin{vmatrix} 0+4785 & 2+4785 \\ 4+4785 & 6+4785 \end{vmatrix}$

Using the property that if some or all elements of a row or column of a determinant are expressed as the sum of two (or more) terms, then the determinant can be expressed as the sum of two (or more) determinants.

We get, $\Delta = \begin{vmatrix} 0 & 2 \\ 4 & 6 \end{vmatrix} + \begin{vmatrix} 4785 & 4785 \\ 4785 & 4785 \end{vmatrix}$

Using the property that If any two rows (or columns) of a determinant are identical (all corresponding elements are same), then the value of the determinant is zero.

Hence, $\Delta = \begin{vmatrix} 0 & 2 \\ 4 & 6 \end{vmatrix} + 0$

=0×6-2×4=-8 (Expanding along R_1)

11. Question

If w is an imaginary cube root of unity, find the value of $\begin{vmatrix} 1 & w & w^2 \\ w & w^2 & 1 \\ w^2 & 1 & w \end{vmatrix}$.

Answer

Let $\Delta = \begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$

Using the property that if the equimultiples of corresponding elements of other rows (or columns) are added to every element of any row (or column) of a determinant, then the value of determinant remains the same

Using row transformation, $R_2 \to R_2 - \omega R_1$

$$\Delta = \begin{vmatrix} 1 & \omega & \omega^2 \\ \omega - \omega & \omega^2 - \omega^2 & 1 - \omega^3 \\ \omega^2 & 1 & \omega \end{vmatrix}$$

$= \begin{vmatrix} 1 & \omega & \omega^2 \\ 0 & 0 & 0 \\ \omega^2 & 1 & \omega \end{vmatrix}$ (Since, ω is a cube root of 1, therefore $\omega^3=1$)

Using the property that if all elements of a row or column of a determinant are 0, the value of determinant is 0.

Hence Δ=0

12. Question

If $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix}$, find $|AB|$.

Answer

Given that $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix}$, we have to find |AB|

Then, $|A| = \begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix}$ and $|B| = \begin{vmatrix} 1 & -4 \\ 3 & -2 \end{vmatrix}$

|A|=1×-1-2×3

=-7 (Expanding along R_1)

|B|=1×(-2)-(-4)×3

=10 (Expanding along R_1)

Since |AB|=|A||B|,

Therefore |AB|=-7×10=-70

13. Question

If A = $[a_{ij}]$ is a 3 × 3 diagonal matrix such that a_{11} = 1, a_{22} = 2 and a_{33} = 3, then find |A|.

Answer

Since A is a diagonal matrix, therefore, all it's non-diagonal members are 0. And $a_{11}=1$, $a_{22}=2$ and $a_{33}=3$

We get $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

Then, $|A| = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{vmatrix}$

Expanding along R_1

$|A| = 1(2\times3-0)=6$

14. Question

If A = $[a_{ij}]$ is a 3 × 3 scalar matrix such that $a_{11} = 2$, then write the value of |A|.

Answer

A scalar matrix is a matrix of order m which is equal to a constant λ multiplied with the Identity matrix of order m.

Since $a_{11}=2$, hence λ=2 and m=3

Hence $A = 2I = 2 \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

Then, $|A| = \begin{vmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{vmatrix}$

Expanding along R_1

$|A|=2(2\times2-0)$

$=8$

15. Question

If I_3 denotes identity matrix of order 3 × 3, write the value of its determinant.

Answer

$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Then, $|I_3| = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

Expanding along R_1

$|I_3|=1(1\times1-0)$

$=1$

16. Question

A matrix A of order 3 × 3 has determinant 5. What is the value of |3A|?

Answer

If the determinant of a matrix A of order m is Δ, then the determinant of matrix λA, where λ is a scalar, is $\lambda^m\Delta$.

In this question, Δ=5, λ=3 and m=3.

$|\lambda A|=3^3\times5$

$=135$

17. Question

On expanding by first row, the value of the determinant of 3 × 3 square matrix A = $[a_{ij}]$ is $a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$, where C_{ij} is the cofactor of a_{ij} is the cofactor of a_{ij} in A. Write the expression for its value of expanding by second column.

Answer

The value of determinant written in the form of cofactors is equal to the sum of products of elements of that row (or column) multiplied by their corresponding cofactors.

Hence, the value of determinant |A|, of matrix $A=[a_{ij}]$ of order 3×3, expanded along column 2 will be

$|A|=a_{12}\times C_{12}+a_{22}\times C_{22}+a_{32}\times C_{32}$

18. Question

Let A = $[a_{ij}]$ be a square matrix of order 3 × 3 and C_{ij} denote cofactor of a_{ij} in A. If |A| = 5, write the value of $a_{31}C_{31} + a_{32}C_{32} + a_{33}C_{33}$.

Answer

The value of determinant |A||, of matrix $A=[a_{ij}]$ of order 3×3, is given to be 5.

The value of determinant written in the form of cofactors is equal to the sum of products of elements of that row (or column) multiplied by their corresponding cofactors.

The value of |A| expanded along row 3 will be

$|A|= a_{31}\times C_{31}+a_{32}\times C_{32}+a_{33}\times C_{33}$, which is the required expression

Hence, the value of required expression is equal to |A|=5.

19. Question

In question 18, write the value of $a_{11}C_{21} + a_{12}C_{22} + a_{13}C_{23}$.

Answer

We have to find out the value of $a_{11}\times C_{21}+a_{12}\times C_{22}+a_{13}\times C_{23}$

Let$I=a_{11}\times C_{21}+a_{12}\times C_{22}+a_{13}\times C_{23}$

$I= a_{11}\times-(a_{12}a_{33}-a_{13}a_{32})+a_{12}\times(a_{11}a_{33}-a_{13}a_{31})+a_{13}\times-(a_{11}a_{32}-a_{12}a_{31})$

$I=-a_{11}a_{12}a_{33}+a_{11}a_{13}a_{32}+a_{11}a_{12}a_{33}-a_{12}a_{13}a_{31}-a_{11}a_{13}a_{32}+a_{12}a_{13}a_{31}$

$I= a_{11}a_{12}a_{33}-a_{11}a_{12}a_{33}+a_{11}a_{13}a_{32}-a_{11}a_{13}a_{32}+ a_{12}a_{13}a_{31}- a_{12}a_{13}a_{31}$

$I=0$

20. Question

Write the value of $\begin{vmatrix} \sin 20^\circ & -\cos 20^\circ \\ \sin 70^\circ & \cos 70^\circ \end{vmatrix}$.

Answer

Let $\Delta=\begin{vmatrix} sin20^\circ & -cos20^\circ \\ sin70^\circ & cos70^\circ \end{vmatrix}$

Expanding along R_1,

we get $\Delta= \sin20^\circ\cos70^\circ-(-\cos20^\circ)\sin70^\circ$

$= \sin20°\cos70°+\cos20°\sin70°$

Since $\sin(A+B)= \sin A\cos B+\cos A\sin B$

Hence, $\sin20°\cos70°+\cos20°\sin70°=\sin(20°+70°)$

$=\sin(90°)$

$=1$

Hence, $\Delta=1$

21. Question

If A is a square matrix satisfying $A^TA = I$, write the value of $|A|$.

Answer

Since $A^TA=I$

Taking determinant both sides

$|A^TA|=|I|$

Using $|AB|=|A||B|$,

$|A^T|=|A|$ and $|I|=1$, we get

$|A||A|=1$

$(|A|)^2=1$

Hence, $|A|=\pm1$

22. Question

If A and B are square matrices of the same order such that $|A| = 3$ and $AB = I$, then write the value of $|B|$.

Answer

Given that $|A|=3$ and $AB=I$

Since $AB=I$

Taking determinant both sides

$|AB|=|I|$

Using $|AB|=|A||B|,|A|=3$ and $|I|=1$, we get

$3|B|=1$

Hence, $|B| = \frac{1}{3}$

23. Question

A is skew-symmetric of order 3, write the value of $|A|$.

Answer

Since A is a skew-symmetric matrix, Therefore

$A^T=-A$

Taking determinant both sides

$|A^T|=|-A|$

Using $|A^T|=|A|$ and $|\lambda A|=\lambda^m|A|$ where m is the order of A

$|A|=(-1)^3|A|$

$=-|A|$ or $2|A|=0$

Hence, $|A|=0$

24. Question

If A is a square matrix of order 3 with determinant 4, then write the value of $|-A|$.

Answer

Since $|\lambda A|= \lambda^m|A|$

Given that $\lambda=-1$, $m=3$ and $|A|=4$, we get

$|-A|=(-1)^3\times 4=-4$

25. Question

If A is a square matrix such that $|A| = 2$, write the value of $|AA^T|$.

Answer

Given that $|A|=2$, we have to find $|AA^T|$

Using $|AB|=|A||B|$ and $|A^T|=|A|$, we get

$|AA^T|=|A||A^T|$

$=|A||A|$

$=2\times 2$

$=4$

26. Question

Find the value of the determinant $\begin{vmatrix} 243 & 156 & 300 \\ 81 & 52 & 100 \\ -3 & 0 & 4 \end{vmatrix}$.

Answer

Let $\Delta=\begin{vmatrix} 243 & 156 & 300 \\ 81 & 52 & 100 \\ -3 & 0 & 4 \end{vmatrix}$

Using the property that if the equimultiples of corresponding elements of other rows (or columns) are added to every element of any row (or column) of a determinant, then the value of determinant remains the same

Using row transformation, $R_1 \to R_1-3R_2$

We get, $\Delta=\begin{vmatrix} 243-81\times 3 & 156-52\times 3 & 300-100\times 3 \\ 81 & 52 & 100 \\ -3 & 0 & 4 \end{vmatrix}$

$=\begin{vmatrix} 0 & 0 & 0 \\ 81 & 52 & 100 \\ -3 & 0 & 4 \end{vmatrix}$

Using the property that if all elements of a row or column of a determinant are 0, the value of determinant is 0.

Hence $\Delta=0$

27. Question

Write the value of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 4 & -6 & 10 \\ 6 & -9 & 15 \end{vmatrix}$.

Answer

Let $\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 4 & -6 & 10 \\ 6 & -9 & 15 \end{vmatrix}$

$= \begin{vmatrix} 2 & -3 & 5 \\ 2 \times 2 & -3 \times 2 & 5 \times 2 \\ 2 \times 3 & -3 \times 3 & 5 \times 3 \end{vmatrix}$

Using the property that if each element of a row (or a column) of a determinant is multiplied by a constant k, then its value gets multiplied by k.

Taking out factor 2 from R_2 and 3 from R_3,

We get, $\Delta = 2 \times 3 \times \begin{vmatrix} 2 & -3 & 5 \\ 2 & -3 & 5 \\ 2 & -3 & 5 \end{vmatrix}$

Using the property that If any two rows (or columns) of a determinant are identical (all corresponding elements are same), then the value of the determinant is zero.

Since R_1, R_2 and R_3 are identical, therefore $\Delta=0$

28. Question

If the matrix $\begin{bmatrix} 5x & 2 \\ -10 & 1 \end{bmatrix}$ is singular, find the value of x.

Answer

Let $A = \begin{bmatrix} 5x & 2 \\ -10 & 1 \end{bmatrix}$

Then, $|A| = \begin{vmatrix} 5x & 2 \\ -10 & 1 \end{vmatrix}$

$=5x \times 1-2 \times -10$ (Expanding along R_1)

$|A|=5x+20$

For A to be singular, $|A|=0$

Hence $5x+20=0$ or $x=-4$

29. Question

If A is a square matrix of order n × n such that $|A| = \lambda$, then write the value of $|-A|$.

Answer

Since $|kA|= k^m|A|$

Given that $k=-1$, $m=n$ and $|A|=\lambda$, we get

$|-A|=(-1)^n \times \lambda$

Hence, $|-A|=\lambda$ if n is even and $|-A|=-\lambda$ if n is odd.

30. Question

Find the value of the determinant $\begin{vmatrix} 2^2 & 2^3 & 2^4 \\ 2^3 & 2^4 & 2^5 \\ 2^4 & 2^5 & 2^4 \end{vmatrix}$.

Answer

Let $\Delta = \begin{vmatrix} 2^2 & 2^3 & 2^4 \\ 2^3 & 2^4 & 2^5 \\ 2^4 & 2^5 & 2^4 \end{vmatrix}$

Using the property that if each element of a row (or a column) of a determinant is multiplied by a constant k, then its value gets multiplied by k.

Taking out factor 2^2 from R_1 and 2^3 from R_2,

$$\Delta = 2^2 \times 2^3 \times \begin{vmatrix} 1 & 2 & 4 \\ 1 & 2 & 4 \\ 2^4 & 2^5 & 2^4 \end{vmatrix}$$

Using the property that If any two rows (or columns) of a determinant are identical (all corresponding elements are same), then the value of the determinant is zero.

Since R_1 and R_2 are identical, therefore $\Delta = 0$.

31. Question

If A and B are non-singular matrices of the same order, write whether AB is singular or non-singular.

Answer

We are given that,

A = non-singular matrix

B = non-singular matrix

Order of A = Order of B

We need to find whether AB is singular or non-singular.

Let us recall the definition of non-singular matrix.

Non-singular matrix, also called regular matrix, is a square matrix that is not singular, i.e., one that has a matrix inverse.

We can say that, a square matrix A is non-singular matrix iff its determinant is non-zero, i.e., $|A| \neq 0$.

While a singular matrix is a square matrix that doesn't have a matrix inverse. Also, the determinant is zero, i.e., $|A| = 0$.

So,

By definition, $|A| \neq 0$ and $|B| \neq 0$ since A and B are non-singular matrices.

Let,

Order of A = Order of B = $n \times n$

⇒ Matrices A and B can be multiplied

⇒ $A \times B = AB$

If we have matrices A and B of same order then we can say that,

$|AB| = 0$ iff either $|A|$ or $|B| = 0$.

And it is clear that, $|A|, |B| \neq 0$.

⇒ $|AB| \neq 0$

And if |AB| ≠ 0, then by definition AB is s non-singular matrix.

Thus, AB is a singular matrix.

32. Question

A matrix of order 3 × 3 has determinant 2. What is the value of |A(3I)|, where I is the identity matrix of order 3 × 3.

Answer

We are given that,

Order of a matrix = 3 × 3

Determinant = 2

I = Identity matrix of order 3 × 3

We need to find the value of |A(3I)|.

Let the given matrix be A.

Then, |A| = 2

Also, since I is an identity matrix, then

$$\det\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix} = 1\big((1\times 1)-(0\times 0)\big)-0\big((0\times 0)+(0\times 1)\big) + 0\big((0\times 0)+(1\times 0)\big)$$

$$\Rightarrow \det\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix} = 1(1-0)-0+0$$

$$\Rightarrow \det\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix} = 1$$

⇒ Det (I) = 1

Or,

|I| = 1

Then, we can say

3(I) = 3

⇒ 3I = 3

Thus,

|A(3I)| = |A(3)| [∵, 3I = 3]

⇒ |A(3I)| = |3A|

By property of determinants, we know that

$|KA| = K^n|A|$, if A is of n^{th} order.

⇒ $|A(3I)| = 3^3|A|$ [∵, A has an order of 3 × 3 ⇒ $|3A| = 3^3\ |A|$]

⇒ |A(3I)| = 27 |A|

Since, |A| = 2. Then,

⇒ |A(3I)| = 27 × 2

⇒ |A(3I)| = 54

Thus, |A(3I)| = 54.

33. Question

If A and B are square matrices of order 3 such that |A| = -1, |B| = 3, then find the value of |3AB|.

Answer

We are given that,

A and B are square matrices of order 3.

|A| = -1, |B| = 3

We need to find the value of |3AB|.

By property of determinant,

$|KA| = K^n|A|$

If A is of n^{th} order.

If order of A = 3 × 3

And order of B = 3 × 3

⇒ Order of AB = 3 × 3 [∵, Number of columns in A = Number of rows in B]

We can write,

$|3AB| = 3^3|AB|$ [∵, Order of AB = 3 × 3]

Now, |AB| = |A||B|.

⇒ |3AB| = 27|A||B|

Putting |A| = -1 and |B| = 3, we get

⇒ |3AB| = 27 × -1 × 3

⇒ |3AB| = -81

Thus, the value of |3AB| = -81.

34. Question

Write the value of $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$.

Answer

We need to find the value of

$$\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$$

Determinant of 2 × 2 matrix is found as,

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = a \times d - b \times c$$

So,

$$\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix} = (a+ib)(a-ib) - (c+id)(-c+id)$$

Rearranging,

$$\Rightarrow \begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix} = (a+ib)(a-ib) - (id+c)(id-c)$$

Using the algebraic identity,

$(x + y)(x - y) = x^2 - y^2$

$\Rightarrow \begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix} = (a^2 - (ib)^2) - ((id)^2 - c^2)$

$\Rightarrow \begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix} = (a^2 - i^2b^2) - (i^2d^2 - c^2)$

$\Rightarrow \begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix} = a^2 - i^2b^2 - i^2d^2 + c^2$

Here, i is iota, an imaginary number.

Note that,

$i^2 = -1$

So,

$\Rightarrow \begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix} = a^2 - (-1)b^2 - (-1)d^2 + c^2$

$\Rightarrow \begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix} = a^2 + b^2 + d^2 + c^2$

Thus,

$\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix} = a^2 + b^2 + c^2 + d^2$

35. Question

Write the cofactor of a_{12} in the matrix $\begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$.

Answer

We need to find the cofactor of a_{12} in the matrix

$\begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$

A cofactor is the number you get when you remove the column and row of a designated element in a matrix, which is just a numerical grid in the form of a rectangle or a square. The cofactor is always preceded by a positive (+) or negative (-) sign, depending whether the element is in a + or - position. It is

$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$

Let us recall how to find the cofactor of any element:

If we are given with,

$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

Cofactor of any element, say a_{11} is found by eliminating first row and first column.

$\text{Cofactor of } a_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$

⇒ Cofactor of $a_{11} = a_{22} \times a_{33} - a_{23} \times a_{32}$

The sign of cofactor of a_{11} is (+).

And, cofactor of any element, say a_{12} is found by eliminating first row and second column.

$$\text{Cofactor of } a_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

⇒ Cofactor of $a_{12} = a_{21} \times a_{33} - a_{23} \times a_{31}$

The sign of cofactor of a_{12} is (-).

Similarly,

First know what the element at position a_{12} in the matrix is.

In $\begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$,

$a_{12} = -3$

And as discussed above, the sign at a_{12} is (-).

For cofactor of -3, eliminate first row and second column in the matrix.

$$\text{Cofactor of } -3 = \begin{vmatrix} 6 & 4 \\ 1 & -7 \end{vmatrix}$$

⇒ Cofactor of -3 = (6 × -7) – (4 × 1)

⇒ Cofactor of -3 = -42 – 4

⇒ Cofactor of -3 = -46

Since, the sign of cofactor of -3 is (-), then

Cofactor of -3 = -(-46)

⇒ Cofactor of -3 = 46

Thus, cofactor of -3 is 46.

36. Question

If $\begin{bmatrix} 2x+5 & 3 \\ 5x+2 & 9 \end{bmatrix} = 0$, find x.

Answer

9(2x + 5) – 3(5x+2) = 0

⇒ 18x + 45 – 15x – 6 = 0

⇒ 3x + 39 = 0

⇒ 3x = - 39

⇒ x = -13

37. Question

Find the value of x from the following: $\begin{vmatrix} x & 2 \\ 2 & 2x \end{vmatrix} = 0$.

Answer

We are given that,

$$\begin{vmatrix} x & 2 \\ 2 & 2x \end{vmatrix} = 0$$

We need to find the value of x.

Determinant of 2 × 2 matrix is found as,

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = a \times d - b \times c$$

So, determinant of the given matrix is found as,

$$\begin{vmatrix} x & 2 \\ 2 & 2x \end{vmatrix} = x \times 2x - 2 \times 2$$

$$\Rightarrow \begin{vmatrix} x & 2 \\ 2 & 2x \end{vmatrix} = 2x^2 - 4$$

According to the question, equate this to 0.

$2x^2 - 4 = 0$

We need to solve the algebraic equation.

$2x^2 = 4$

$$\Rightarrow x^2 = \frac{4}{2}$$

$\Rightarrow x^2 = 2$

Taking square root on both sides of the equation,

$$\Rightarrow \sqrt{x^2} = \pm\sqrt{2}$$

$\Rightarrow x = \pm\sqrt{2}$

Hence, the value of x is $\pm\sqrt{2}$.

38. Question

Write the value of the determinant $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$.

Answer

We need to find the value of determinant,

$$\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$$

Determinant of 3 × 3 matrices is found as,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}.\det\begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} - a_{12}.\det\begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix} + a_{13}.\det\begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

$$\Rightarrow \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}(a_{22} \times a_{33} - a_{23} \times a_{32}) - a_{12}(a_{21} \times a_{33} - a_{23} \times a_{31}) + a_{13}(a_{21} \times a_{32} - a_{22} \times a_{31})$$

Similarly,

$$\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix} = 2.\det\begin{bmatrix} 6 & 8 \\ 9x & 12x \end{bmatrix} - 3.\det\begin{bmatrix} 5 & 8 \\ 6x & 12x \end{bmatrix} + 4.\det\begin{bmatrix} 5 & 6 \\ 6x & 9x \end{bmatrix}$$

$$\Rightarrow \begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$$
$$= 2(6 \times 12x - 8 \times 9x) - 3(5 \times 12x - 8 \times 6x) + 4(5 \times 9x - 6 \times 6x)$$

$$\Rightarrow \begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix} = 2(72x - 72x) - 3(60x - 48x) + 4(45x - 36x)$$

$$\Rightarrow \begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix} = 2(0) - 3(12x) + 4(9x)$$

$$\Rightarrow \begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix} = 0 - 36x + 36x$$

$$\Rightarrow \begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix} = 0$$

Thus, the value of $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix} = 0$.

39. Question

If |A| = 2, where A is 2 × 2 matrix, find |adj A|.

Answer

We are given that,

Order of matrix A = 2 × 2

|A| = 2

We need to find the |adj A|.

Let us understand what adjoint of a matrix is.

Let A = $[a_{ij}]$ be a square matrix of order n × n. Then, the adjoint of the matrix A is transpose of the cofactor of matrix A.

The relationship between adjoint of matrix and determinant of matrix is given as,

$|adj\ A| = |A|^{n-1}$

Where, n = order of the matrix

Putting |A| = 2 in the above equation,

⇒ $|adj\ A| = (2)^{n-1}$ …(i)

Here, order of matrix A = 2

∴, n = 2

Putting n = 2 in equation (i), we get

⇒ $|adj\ A| = (2)^{2-1}$

⇒ $|adj\ A| = (2)^{1}$

⇒ |adj A| = 2

Thus, the |adj A| is 2.

40. Question

For what is the value of the determinant $\begin{vmatrix} 0 & 2 & 0 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{vmatrix}$?

Answer

We need to find the value of determinant,

$$\begin{vmatrix} 0 & 2 & 0 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{vmatrix}$$

Determinant of 3 × 3 matrices is found as,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}.\det\begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} - a_{12}.\det\begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix} + a_{13}.\det\begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

$$\Rightarrow \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}(a_{22} \times a_{33} - a_{23} \times a_{32}) - a_{12}(a_{21} \times a_{33} - a_{23} \times a_{31}) + a_{13}(a_{21} \times a_{32} - a_{22} \times a_{31})$$

Similarly,

$$\begin{vmatrix} 0 & 2 & 0 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{vmatrix} = 0.\det\begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix} - 2.\det\begin{bmatrix} 2 & 4 \\ 4 & 6 \end{bmatrix} - 0.\det\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{vmatrix} 0 & 2 & 0 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{vmatrix} = 0(3 \times 6 - 4 \times 5) - 2(2 \times 6 - 4 \times 4) - 0(2 \times 5 - 3 \times 4)$$

$$\Rightarrow \begin{vmatrix} 0 & 2 & 0 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{vmatrix} = 0(18 - 20) - 2(12 - 16) - 0(10 - 12)$$

$$\Rightarrow \begin{vmatrix} 0 & 2 & 0 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{vmatrix} = 0(-2) - 2(-4) - 0(-2)$$

$$\Rightarrow \begin{vmatrix} 0 & 2 & 0 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{vmatrix} = 0 + 8 + 0$$

$$\Rightarrow \begin{vmatrix} 0 & 2 & 0 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{vmatrix} = 8$$

Thus, the value of $\begin{vmatrix} 0 & 2 & 0 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{vmatrix}$ is 8.

41. Question

For what value of x is the matrix $\begin{bmatrix} 6-x & 4 \\ 3-x & 1 \end{bmatrix}$ singular?

Answer

We are given that,

$\begin{bmatrix} 6-x & 4 \\ 3-x & 1 \end{bmatrix}$ is singular matrix.

We need to find the value of x.

Let us recall the definition of singular matrix.

A singular matrix is a square matrix that doesn’t have a matrix inverse. A matrix ‘A’ is singular iff its determinant is zero, i.e., |A| = 0.

Hence, we just need to find the determinant of the given matrix and equate it to zero.

Determinant of 2 × 2 matrix is found as,

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = a \times d - b \times c$$

So,

$$\begin{vmatrix} 6-x & 4 \\ 3-x & 1 \end{vmatrix} = (6-x) \times 1 - 4 \times (3-x)$$

$$\Rightarrow \begin{vmatrix} 6-x & 4 \\ 3-x & 1 \end{vmatrix} = (6-x) - (12-4x)$$

$$\Rightarrow \begin{vmatrix} 6-x & 4 \\ 3-x & 1 \end{vmatrix} = 6 - x - 12 + 4x$$

$$\Rightarrow \begin{vmatrix} 6-x & 4 \\ 3-x & 1 \end{vmatrix} = 4x - x - 12 + 6$$

$$\Rightarrow \begin{vmatrix} 6-x & 4 \\ 3-x & 1 \end{vmatrix} = 3x - 6$$

Now, equate this to 0.

That is,

$$\begin{vmatrix} 6-x & 4 \\ 3-x & 1 \end{vmatrix} = 0$$

⇒ 3x - 6 = 0

⇒ 3x = 6

$$\Rightarrow x = \frac{6}{3}$$

⇒ x = 2

Thus, the value of x = 2 for which the matrix is singular.

42. Question

A matrix A of order 3 × 3 is such that |A| = 4. Find the value of |2A|.

Answer

We are given that,

Order of matrix A = 3

|A| = 4

We need to find the value of |2A|.

By property of determinant of matrix,

$|KA| = K^n|A|$

Where, order of the matrix A is n.

Similarly,

$|2A| = 2^3|A|$

[∵, Order of matrix A = 3]

⇒ $|2A| = 8|A|$

Substituting the value of |A| in the above equation,

⇒ $|2A| = 8 \times 4$

⇒ $|2A| = 32$

Thus, the value of |2A| is 32.

43. Question

Evaluate: $\begin{vmatrix} \cos 15° & \sin 15° \\ \sin 75° & \cos 75° \end{vmatrix}$

Answer

We need to evaluate the matrix:

$$\begin{vmatrix} \cos 15° & \sin 15° \\ \sin 75° & \cos 75° \end{vmatrix}$$

Determinant of 2 × 2 matrix is found as,

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = a \times d - b \times c$$

So,

$$\begin{vmatrix} \cos 15° & \sin 15° \\ \sin 75° & \cos 75° \end{vmatrix} = \cos 15° \times \cos 75° - \sin 15° \times \sin 75°$$

Using the trigonometric identity,

cos (A + B) = cos A cos B – sin A sin B

Replace A by 15° and B by 75°.

cos (15° + 75°) = cos 15° cos 75° - sin 15° cos 75°

⇒ cos 90° = cos 15° cos 75° - sin 15° cos 75°

So, substituting it, we get

$$\Rightarrow \begin{vmatrix} \cos 15° & \sin 15° \\ \sin 75° & \cos 75° \end{vmatrix} = \cos 90°$$

Using the trigonometric identity,

cos 90° = 0

$$\Rightarrow \begin{vmatrix} \cos 15° & \sin 15° \\ \sin 75° & \cos 75° \end{vmatrix} = 0$$

Thus, the value of $\begin{vmatrix} \cos 15° & \sin 15° \\ \sin 75° & \cos 75° \end{vmatrix} = 0$.

44. Question

If $A = \begin{bmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix}$. Write the cofactor of the element a_{32}.

Answer

We are given that,

$$A = \begin{bmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

We need to find the cofactor of the element a_{32}.

A cofactor is the number you get when you remove the column and row of a designated element in a matrix, which is just a numerical grid in the form of a rectangle or a square. The cofactor is always preceded by a positive (+) or negative (-) sign, depending whether the element is in a + or - position. It is

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

Let us recall how to find the cofactor of any element:

If we are given with,

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Cofactor of any element, say a_{11} is found by eliminating first row and first column.

$$\text{Cofactor of } a_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$$

⇒ Cofactor of $a_{11} = a_{22} \times a_{33} - a_{23} \times a_{32}$

The sign of cofactor of a_{11} is (+).

And, cofactor of any element, say a_{12} is found by eliminating first row and second column.

$$\text{Cofactor of } a_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

⇒ Cofactor of $a_{12} = a_{21} \times a_{33} - a_{23} \times a_{31}$

The sign of cofactor of a_{12} is (-).

So,

In matrix, $A = \begin{bmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix}$.

Element at $a_{32} = 2$

We need to find the cofactor of 2 at a_{32}.

And as discussed above, the sign at a_{32} is (-).

For cofactor of a_{32}, eliminate third row and second column in the matrix.

$$\text{Cofactor of } a_{32} = \begin{vmatrix} 5 & 8 \\ 2 & 1 \end{vmatrix}$$

⇒ Cofactor of $a_{32} = 5 \times 1 - 8 \times 2$

⇒ Cofactor of $a_{32} = 5 - 16$

⇒ Cofactor of $a_{32} = -11$

Since, the sign of cofactor of a_{32} is (-), then

Cofactor of $a_{32} = -(-11)$

⇒ Cofactor of $a_{32} = 11$

Thus, cofactor of a_{32} is 11.

45. Question

If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$, then write the value of x.

Answer

We are given that,

$$\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$$

We need to find the value of x.

Determinant of 2 × 2 matrix is found as,

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = a \times d - b \times c$$

Let us take left hand side (LHS) of the given matrix equation.

$$\text{LHS} = \begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix}$$

⇒ LHS = (x + 1)(x + 2) – (x – 1)(x – 3)

⇒ LHS = (x^2 + x + 2x + 2) – (x^2 – x – 3x + 3)

⇒ LHS = (x^2 + 3x + 2) – (x^2 – 4x + 3)

⇒ LHS = x^2 + 3x + 2 – x^2 + 4x – 3

⇒ LHS = x^2 – x^2 + 3x + 4x + 2 – 3

⇒ LHS = 7x – 1

Let us take right hand side (RHS) of the given matrix equation.

$$\text{RHS} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$$

⇒ RHS = 4 × 3 – (-1) × 1

⇒ RHS = 12 + 1

⇒ RHS = 13

Now,

LHS = RHS

⇒ 7x – 1 = 13

⇒ 7x = 13 + 1

⇒ 7x = 14

$$\Rightarrow x = \frac{14}{7}$$

⇒ x = 2

Thus, the value of x is 2.

46. Question

If $\begin{vmatrix} 2x & x+3 \\ 2(x+1) & x+1 \end{vmatrix} = \begin{vmatrix} 1 & 5 \\ 3 & 3 \end{vmatrix}$, then write the value of x.

Answer

We are given that,

$$\begin{vmatrix} 2x & x+3 \\ 2(x+1) & x+1 \end{vmatrix} = \begin{vmatrix} 1 & 5 \\ 3 & 3 \end{vmatrix}$$

We need to find the value of x.

Determinant of 2 × 2 matrix is found as,

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = a \times d - b \times c$$

Let us take left hand side (LHS) of the given matrix equation.

$$\text{LHS} = \begin{vmatrix} 2x & x+3 \\ 2(x+1) & x+1 \end{vmatrix}$$

⇒ LHS = 2x(x + 1) – (x + 3)(2(x + 1))

⇒ LHS = $(2x^2 + 2x) - (x + 3)(2x + 2)$

⇒ LHS = $(2x^2 + 2x) - (2x^2 + 2x + 6x + 6)$

⇒ LHS = $(2x^2 + 2x) - (2x^2 + 8x + 6)$

⇒ LHS = $2x^2 + 2x - 2x^2 - 8x - 6$

⇒ LHS = $2x^2 - 2x^2 + 2x - 8x - 6$

⇒ LHS = -6x – 6

Let us take right hand side (RHS) of the given matrix equation.

$$\text{RHS} = \begin{vmatrix} 1 & 5 \\ 3 & 3 \end{vmatrix}$$

⇒ RHS = 1 × 3 – 5 × 3

⇒ RHS = 3 – 15

⇒ RHS = -12

Now,

LHS = RHS

⇒ -6x – 6 = -12

⇒ -6x = -12 + 6

⇒ -6x = -6

$$\Rightarrow x = \frac{-6}{-6}$$

⇒ x = 1

Thus, the value of x is 1.

47. Question

If $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$, find the value of x.

Answer

We are given that,

$\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$

We need to find the value of x.

Determinant of 2 × 2 matrix is found as,

$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = a \times d - b \times c$

Let us take left hand side (LHS) of the given matrix equation.

$\text{LHS} = \begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix}$

⇒ LHS = 3x × 4 – 7 × (-2)

⇒ LHS = 12x – (-14)

⇒ LHS = 12x + 14

Let us take right hand side (RHS) of the given matrix equation.

$\text{RHS} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$

⇒ RHS = 8 × 4 – 7 × 6

⇒ RHS = 32 – 42

⇒ RHS = -10

Now,

LHS = RHS

⇒ 12x + 14 = -10

⇒ 12x = -10 – 14

⇒ 12x = -24

$\Rightarrow x = \frac{-24}{12}$

⇒ x = -2

Thus, the value of x is -2.

48. Question

If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, write the value of x.

Answer

We are given that,

$\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$

We need to find the value of x.

Determinant of 2 × 2 matrix is found as,

$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = a \times d - b \times c$

Let us take left hand side (LHS) of the given matrix equation.

$\text{LHS} = \begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix}$

$\Rightarrow$ LHS = 2x × x – 5 × 8

$\Rightarrow$ LHS = $2x^2 - 40$

Let us take right hand side (RHS) of the given matrix equation.

$$\text{RHS} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$$

$\Rightarrow$ RHS = 6 × 3 – (-2) × 7

$\Rightarrow$ RHS = 18 – (-14)

$\Rightarrow$ RHS = 18 + 14

$\Rightarrow$ RHS = 32

Now,

LHS = RHS

$\Rightarrow 2x^2 - 40 = 32$

$\Rightarrow 2x^2 = 32 + 40$

$\Rightarrow 2x^2 = 72$

$$\Rightarrow x^2 = \frac{72}{2}$$

$\Rightarrow x^2 = 36$

$\Rightarrow x = \pm\sqrt{36}$

$\Rightarrow x = \pm 6$

Thus, the value of x is ±6.

49. Question

If A is a 3 × 3 matrix, |A| ≠ 0 and |3A| = k|A| then write value of k.

Answer

We are given that,

Order of matrix = 3

|A| ≠ 0

|3A| = k|A|

We need to find the value of k.

In order to find k, we need to solve |3A|.

Using property of determinants,

$|kA| = k^n|A|$

Where, order of A is n × n.

Similarly,

$|3A| = 3^3|A|$

[∵, order of A is 3]

$\Rightarrow$ |3A| = 27|A| …(i)

As, according to the question

|3A| = k|A|

Using (i),

⇒ 27|A| = k|A|

Comparing the left hand side and right hand side, we get

k = 27

Thus, the value of k is 27.

50. Question

Write the value of the determinant $\begin{vmatrix} p & p+1 \\ p-1 & p \end{vmatrix}$.

Answer

We need to find the determinant,

$$\begin{vmatrix} p & p+1 \\ p-1 & p \end{vmatrix}$$

Determinant of 2 × 2 matrix is found as,

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = a \times d - b \times c$$

So,

$$\begin{vmatrix} p & p+1 \\ p-1 & p \end{vmatrix} = p \times p - (p+1)(p-1)$$

Using the algebraic identity,

$(a + b)(a - b) = a^2 - b^2$

$$\Rightarrow \begin{vmatrix} p & p+1 \\ p-1 & p \end{vmatrix} = p^2 - (p^2 - 1)$$

$$\Rightarrow \begin{vmatrix} p & p+1 \\ p-1 & p \end{vmatrix} = p^2 - p^2 + 1$$

$$\Rightarrow \begin{vmatrix} p & p+1 \\ p-1 & p \end{vmatrix} = 1$$

Thus, the value of $\begin{vmatrix} p & p+1 \\ p-1 & p \end{vmatrix} = 1$.

51. Question

Write the value of the determinant $\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix}$.

Answer

We need to find the value of determinant

$$\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix}$$

Determinant of 3 × 3 matrices is found as,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}.\det\begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} - a_{12}.\det\begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix} + a_{13}.\det\begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

$$\Rightarrow \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}(a_{22} \times a_{33} - a_{23} \times a_{32}) - a_{12}(a_{21} \times a_{33} - a_{23} \times a_{31}) + a_{13}(a_{21} \times a_{32} - a_{22} \times a_{31})$$

So,

$$\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix} = (x+y).\det\begin{bmatrix} x & y \\ -3 & -3 \end{bmatrix} - (y+z).\det\begin{bmatrix} z & y \\ -3 & -3 \end{bmatrix} + (z+x).\det\begin{bmatrix} z & x \\ -3 & -3 \end{bmatrix}$$

$$\Rightarrow \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix} = (x+y).\left(x \times (-3) - y \times (-3)\right) - (y+z).\left(z \times (-3) - y \times (-3)\right) + (z+x).\left(z \times (-3) - x \times (-3)\right)$$

$$\Rightarrow \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix} = (x+y)(-3x+3y) - (y+z)(-3z+3y) + (z+x)(-3z+3x)$$

$$\Rightarrow \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix} = 3(x+y)(-x+y) - 3(y+z)(-z+y) + 3(z+x)(-z+x)$$

Re-arranging the equation,

$$\Rightarrow \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix} = 3(y+x)(y-x) - 3(y+z)(y-z) + 3(x+z)(x-z)$$

$$\Rightarrow \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix} = 3[(y+x)(y-x) - (y+z)(y-z) + (x+z)(x-z)]$$

Using the algebraic identity,

$(a + b)(a - b) = a^2 - b^2$

$$\Rightarrow \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix} = 3[(y^2 - x^2) - (y^2 - z^2) + (x^2 - z^2)]$$

$$\Rightarrow \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix} = 3(y^2 - x^2 - y^2 + z^2 + x^2 - z^2)$$

$$\Rightarrow \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix} = 3(x^2 - x^2 + y^2 - y^2 + z^2 - z^2)$$

$$\Rightarrow \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix} = 3(0+0+0)$$

$$\Rightarrow \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix} = 0$$

Thus, the value of $\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix}$ is 0.

52. Question

If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then for any natural number, find the value of Det(A^n).

Answer

We are given that,

$$A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

We need to find the det(A^n).

To find det(A^n),

First we need to find A^n, and then take determinant of A^n.

Let us find A^2.

A^2 = A.A

$$\Rightarrow A^2 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

Let,

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$$

For z_{11}: Dot multiply the first row of the first matrix and first column of the second matrix, then sum up.

That is,

(cos θ, sin θ).(cos θ, -sin θ) = cos θ × cos θ + sin θ × (-sin θ)

⇒ (cos θ, sin θ).(cos θ, -sin θ) = $\cos^2$ θ – $\sin^2$ θ

By algebraic identity,

cos 2θ = $\cos^2$ θ – $\sin^2$ θ

⇒ (cos θ, sin θ).(cos θ, -sin θ) = cos 2θ

$$\Rightarrow \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos 2\theta & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$$

For z_{12}: Dot multiply the first row of the first matrix and second column of the second matrix, then sum up.

That is,

(cos θ, sin θ)(sin θ, cos θ) = cos θ × sin θ + sin θ × cos θ

⇒ (cos θ, sin θ)(sin θ, cos θ) = sin θ cos θ + sin θ cos θ

⇒ (cos θ, sin θ)(sin θ, cos θ) = 2 sin θ cos θ

By algebraic identity,

sin 2θ = 2 sin θ cos θ

⇒ (cos θ, sin θ)(sin θ, cos θ) = sin 2θ

$$\Rightarrow \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ z_{21} & z_{22} \end{bmatrix}$$

Similarly,

$$\Rightarrow \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$
$$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ (-\sin\theta \times \cos\theta) + (\cos\theta \times -\sin\theta) & (-\sin\theta \times \sin\theta) + (\cos\theta \times \cos\theta) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$
$$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin\theta\cos\theta - \sin\theta\cos\theta & -\sin^2\theta + \cos^2\theta \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -2\sin\theta\cos\theta & \cos 2\theta \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ and $A^2 = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$, then

$$A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$$

Now, taking determinant of A^n,

$$\mathrm{Det}(A^n) = \begin{vmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{vmatrix}$$

Determinant of 2 × 2 matrix is found as,

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = a \times d - b \times c$$

So,

$\mathrm{Det}(A^n) = \cos n\theta \times \cos n\theta - \sin n\theta \times (-\sin n\theta)$

$\Rightarrow \mathrm{Det}(A^n) = \cos^2 n\theta + \sin^2 n\theta$

Using the algebraic identity,

$\sin^2 A + \cos^2 A = 1$

$\Rightarrow \mathrm{Det}(A^n) = 1$

Thus, $\mathrm{Det}(A^n)$ is 1.

53. Question

Find the maximum value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 2 & 1+\cos\theta \end{vmatrix}$.

Answer

We need to find the maximum value of

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 2 & 1+\cos\theta \end{vmatrix}$$

Let us find the determinant,

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 2 & 1+\cos\theta \end{vmatrix}$$

Determinant of 3 × 3 matrices is found as,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}.\det\begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} - a_{12}.\det\begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix} + a_{13}.\det\begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

$$\Rightarrow \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}(a_{22} \times a_{33} - a_{23} \times a_{32}) - a_{12}(a_{21} \times a_{33} - a_{23} \times a_{31}) + a_{13}(a_{21} \times a_{32} - a_{22} \times a_{31})$$

So,

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 2 & 1+\cos\theta \end{vmatrix} = 1.\det\begin{bmatrix} 1+\sin\theta & 1 \\ 2 & 1+\cos\theta \end{bmatrix} - 1.\det\begin{bmatrix} 1 & 1 \\ 1 & 1+\cos\theta \end{bmatrix} + 1.\det\begin{bmatrix} 1 & 1+\sin\theta \\ 1 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 2 & 1+\cos\theta \end{vmatrix} = [(1+\sin\theta)(1+\cos\theta) - 1 \times 2] - [1(1+\cos\theta) - 1] + [1 \times 2 - (1+\sin\theta)]$$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 2 & 1+\cos\theta \end{vmatrix} = [1+\cos\theta+\sin\theta+\sin\theta\cos\theta - 2] - [1+\cos\theta - 1] + [2 - 1 - \sin\theta]$$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 2 & 1+\cos\theta \end{vmatrix} = 1+\cos\theta+\sin\theta+\sin\theta\cos\theta - 2 - \cos\theta + 2 - 1 - \sin\theta$$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 2 & 1+\cos\theta \end{vmatrix} = 1 - 2 + 2 - 1 + \sin\theta - \sin\theta + \cos\theta - \cos\theta + \sin\theta\cos\theta$$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 2 & 1+\cos\theta \end{vmatrix} = \sin\theta\cos\theta$$

Multiply and divide by 2 on right hand side,

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 2 & 1+\cos\theta \end{vmatrix} = \frac{2}{2}\sin\theta\cos\theta$$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 2 & 1+\cos\theta \end{vmatrix} = \frac{\sin 2\theta}{2}$$

[∵, By trigonometric identity, sin 2θ = 2 sin θ cos θ]

We need to find the maximum value of $\frac{\sin 2\theta}{2}$.

We know the range of sine function.

-1 ≤ sin A ≤ 1

Or,

-1 ≤ sin 2θ ≤ 1

∴, maximum value of sin 2θ is 1.

⇒ maximum value of $\frac{\sin 2\theta}{2}$ = 1/2

Thus, maximum value of

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 2 & 1+\cos\theta \end{vmatrix} = \frac{1}{2}$$

54. Question

If x ∈ N and $\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8$, then find the value of x.

Answer

We are given that,

$$\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8$$

x ∈ N

We need to find the value of x.

Determinant of 2 × 2 matrix is found as,

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = a \times d - b \times c$$

So, take

$$\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = [(x+3) \times 2x] - [(-2) \times (-3x)]$$

$$\Rightarrow \begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 2x^2 + 6x - 6x$$

$$\Rightarrow \begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 2x^2$$

Since,

$$\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8$$

⇒ $2x^2 = 8$

$$\Rightarrow x^2 = \frac{8}{2}$$

⇒ $x^2 = 4$

⇒ x = ±√4

⇒ x = ±2

Since, x ∈ N

-2 is not a natural number.

Thus, the value of x is 2.

55. Question

If $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} = 8$, write the value of x.

Answer

We are given that,

$$\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} = 8$$

We need to find the value of x.

Determinant of 3 × 3 matrices is found as,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}.\det\begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} - a_{12}.\det\begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix} + a_{13}.\det\begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

$$\Rightarrow \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}(a_{22} \times a_{33} - a_{23} \times a_{32}) - a_{12}(a_{21} \times a_{33} - a_{23} \times a_{31}) + a_{13}(a_{21} \times a_{32} - a_{22} \times a_{31})$$

So,

$$\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} = x.\det\begin{bmatrix} -x & 1 \\ 1 & x \end{bmatrix} - \sin\theta.\det\begin{bmatrix} -\sin\theta & 1 \\ \cos\theta & x \end{bmatrix} + \cos\theta.\det\begin{bmatrix} -\sin\theta & -x \\ \cos\theta & 1 \end{bmatrix}$$

$$\Rightarrow \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} = x.[-x \times x - 1] - \sin\theta.[-\sin\theta \times x - \cos\theta] + \cos\theta.[-\sin\theta - (-x) \times \cos\theta]$$

$$\Rightarrow \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} = x[-x^2 - 1] - \sin\theta[-x\sin\theta - \cos\theta] + \cos\theta[-\sin\theta + x\cos\theta]$$

$$\Rightarrow \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} = -x^3 - x + x\sin^2\theta + \sin\theta\cos\theta - \sin\theta\cos\theta + x\cos^2\theta$$

$$\Rightarrow \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} = -x^3 - x + x\sin^2\theta + x\cos^2\theta + \sin\theta\cos\theta - \sin\theta\cos\theta$$

$$\Rightarrow \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} = -x^3 - x + x(\sin^2\theta + \cos^2\theta)$$

Using trigonometric identity,

$\sin^2 \theta + \cos^2 \theta = 1$

$$\Rightarrow \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} = -x^3 - x + x$$

$$\Rightarrow \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} = -x^3$$

Since,

$$\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} = 8$$

$\Rightarrow -x^3 = 8$

$\Rightarrow x^3 = -8$

$\Rightarrow x^3 = -2 \times -2 \times -2$

Taking cube root on both sides,

$\Rightarrow \sqrt[3]{x^3} = \sqrt[3]{-2 \times -2 \times -2}$

$\Rightarrow x = -2$

Thus, the value of x is -2.

56. Question

If A is a 3 × 3 matrix, then what will be the value of k if $\text{Det}(A^{-1}) = (\text{Det } A)^k$?

Answer

We are given that,

Order of matrix = 3 × 3

$\text{Det}(A^{-1}) = (\text{Det } A)^k$

An n-by-n square matrix A is called invertible if there exists an n-by-n square matrix B such that where I_n denotes the n-by-n identity matrix and the multiplication used is ordinary matrix multiplication.

We know that,

If A and B are square matrices of same order, then

Det (AB) = Det (A).Det (B)

Since, A is an invertible matrix, this means that, A has an inverse called A^{-1}.

Then, if A and A^{-1} are inverse matrices, then

$\text{Det }(AA^{-1}) = \text{Det }(A).\text{Det }(A^{-1})$

By property of inverse matrices,

$AA^{-1} = I$

∴, $\text{Det }(I) = \text{Det }(A).\text{Det }(A^{-1})$

Since, Det (I) = 1

$\Rightarrow 1 = \text{Det }(A).\text{Det }(A^{-1})$

$$\Rightarrow \text{Det}\,(A^{-1}) = \frac{1}{\text{Det}\,(A)}$$

$\Rightarrow$ Det (A^{-1}) = Det $(A)^{-1}$

Since, according to question,

Det(A^{-1}) = (Det $A)^k$

$\Rightarrow k = -1$

Thus, the value of k is -1.